Visualizing Mediation Effects

Applying data ellipses, confidence ellipses and just plain data analysis

Michael Friendly

SCS Seminar, Nov 4, 2011

What is a mediation model?

Seeks to sort out the process whereby a predictor, X, influences a response, Y, by considering one (or more) mediating variables, M.

Emphasis here is on testing significance of various paths:
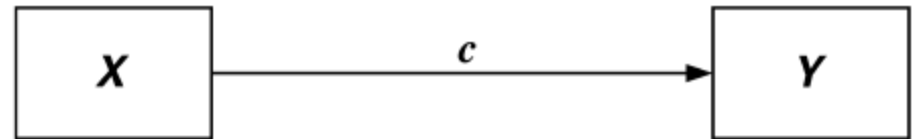
c: direct effect (ignoring M)

c': conditional effect

ab: indirect effect thru M

→ Conclude that M doesn't "explain" relation between X,Y if
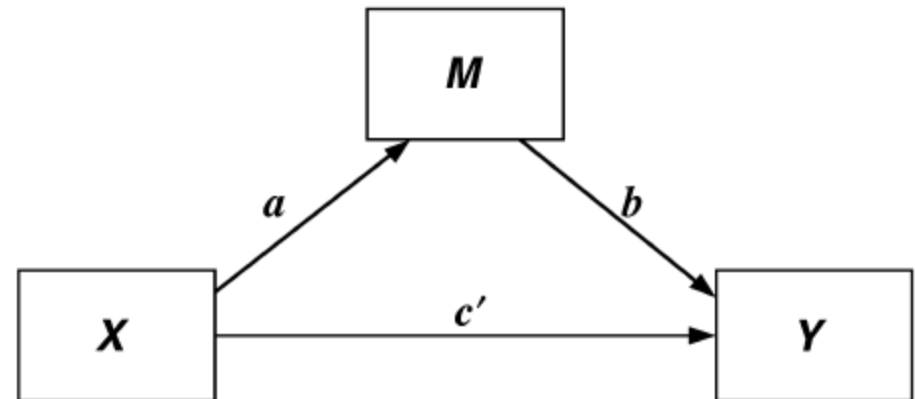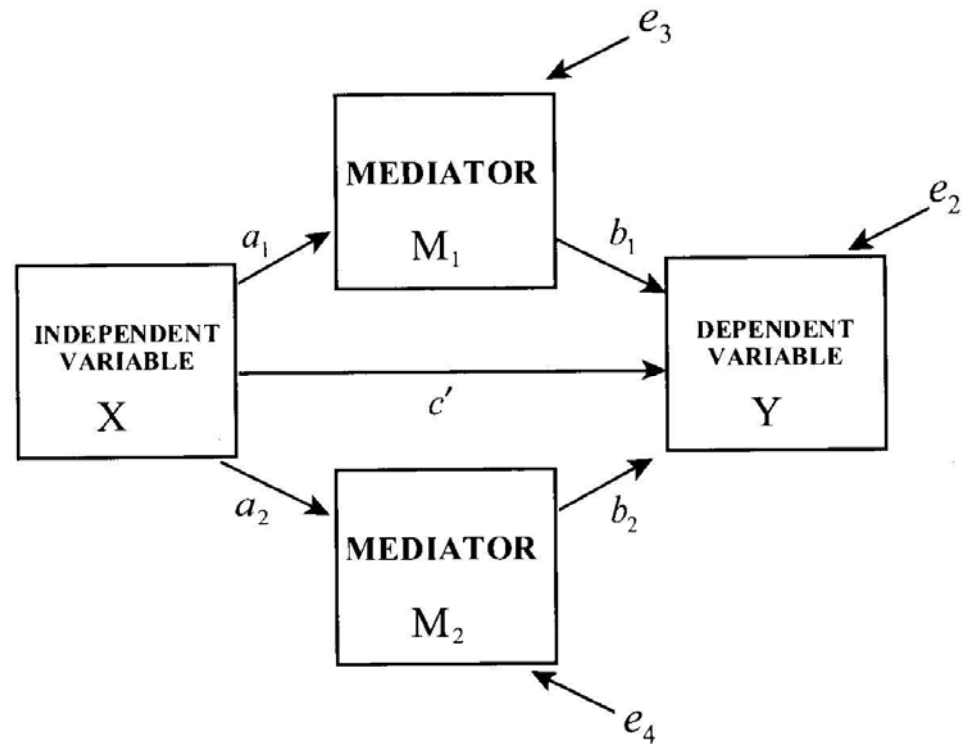
$$ab = (c-c') \approx 0$$

Panel A

Panel B

Figure 1. Panel A: Illustration of a direct effect. *X* affects *Y*. **Panel B: Illustration of a mediation design.** *X* affects *Y* indirectly through *M*.

More complex models

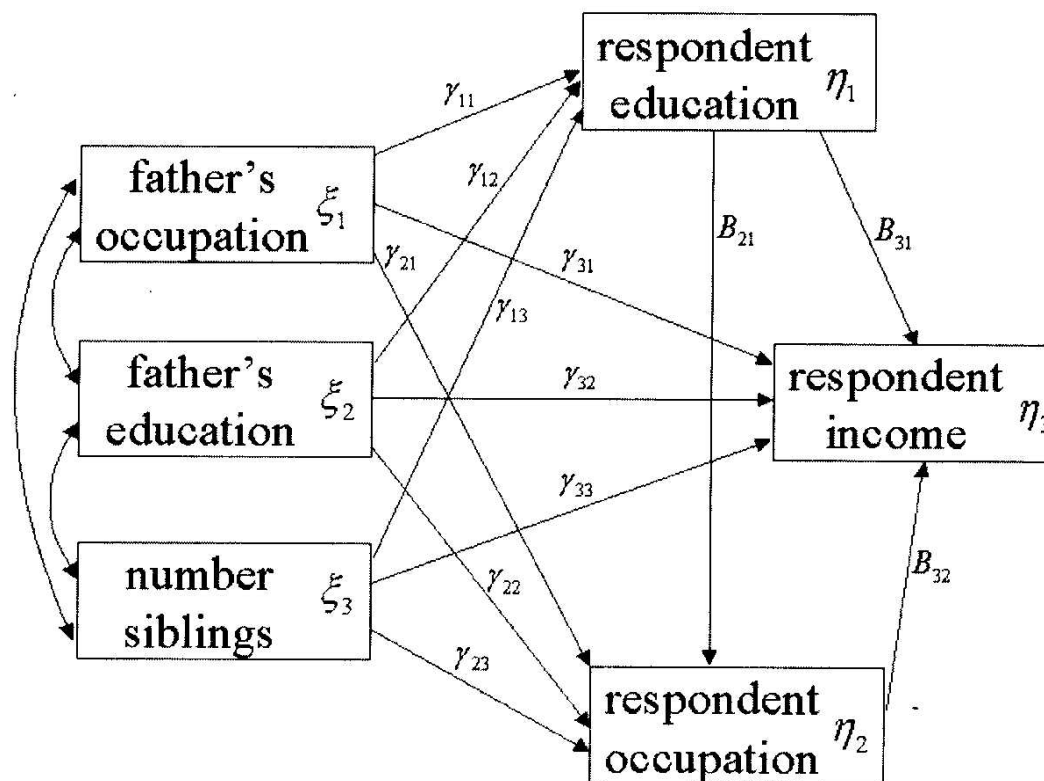There can be multiple mediators, each with indirect paths

More complex models

Path models allow multiple Xs, and multiple mediators (Ms), perhaps with relation among the Ms

SEM software allows calculation of all direct and indirect effects

Various calculators allow testing for specific mediation effects

What's not to like?

What's wrong with these pictures?

- Undue focus on testing significance of coefficients
- All assume that the data are adequately represented by the model
  - relations are linear
  - constant conditional variance
  - no outliers
- → Standard analysis should include graphical displays for the above

Example

Does thirst (M) mediate the relation between temperature (X) and amount of water (Y) drunk?

(Fake data, nothing really surprising)

temperature — X —c→ water — Y

thirst — M

temperature — X, a → M, b → water — Y, c'
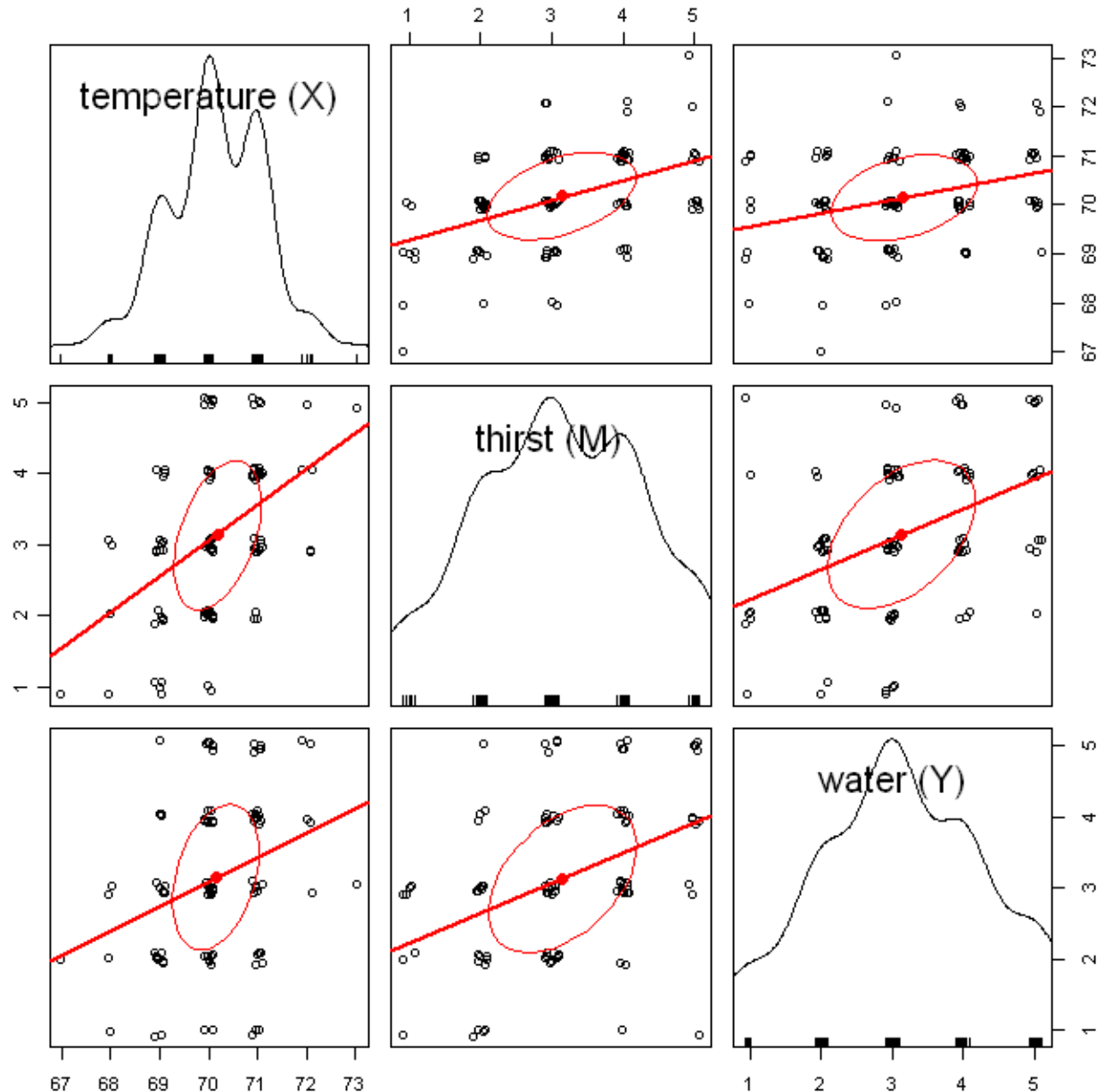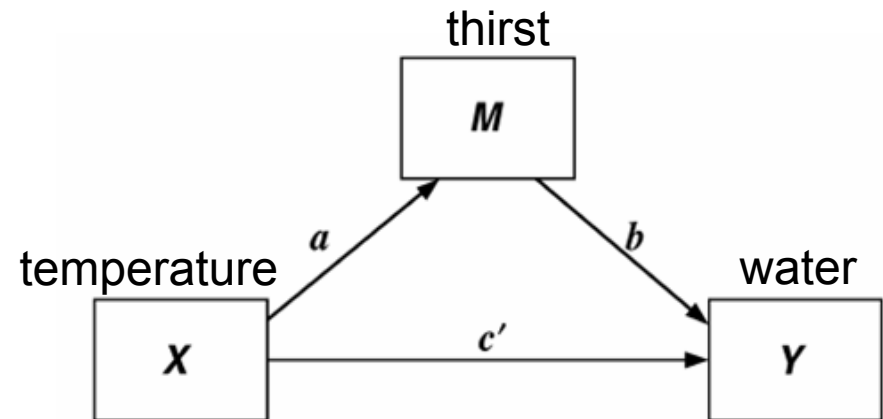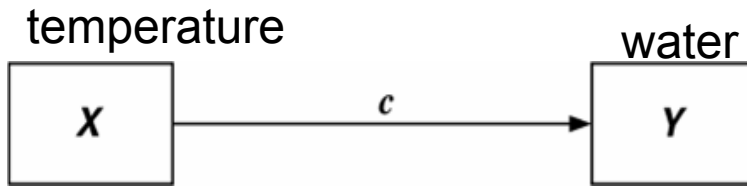
Some numerical output:

```
> library(MBESS)
> mstats <- with(water, mediation(temperature, thirst, water))
> mstats$Y.on.X$Regression.Table
                Estimate Std. Error t value  p(>|t|) Low Conf Limit Up Conf Limit
Intercept.Y_X  -20.8395     7.4829  -2.785 0.006427       -35.6891       -5.9899
c (Regressor)    0.3418      0.1067   3.204 0.001832         0.1301        0.5535


> mstats$Y.on.X.and.M$Regression.Table
                  Estimate Std. Error t value   p(>|t|) Low Conf Limit Up Conf Limit
Intercept.Y_XM     -9.3913     7.7734  -1.208 0.2299348      -24.81936        6.0368
c.prime (Regressor) 0.1624     0.1128   1.440 0.1530583       -0.06143        0.3863
b (Mediator)        0.3611     0.1016   3.554 0.0005873        0.15948        0.5628
```
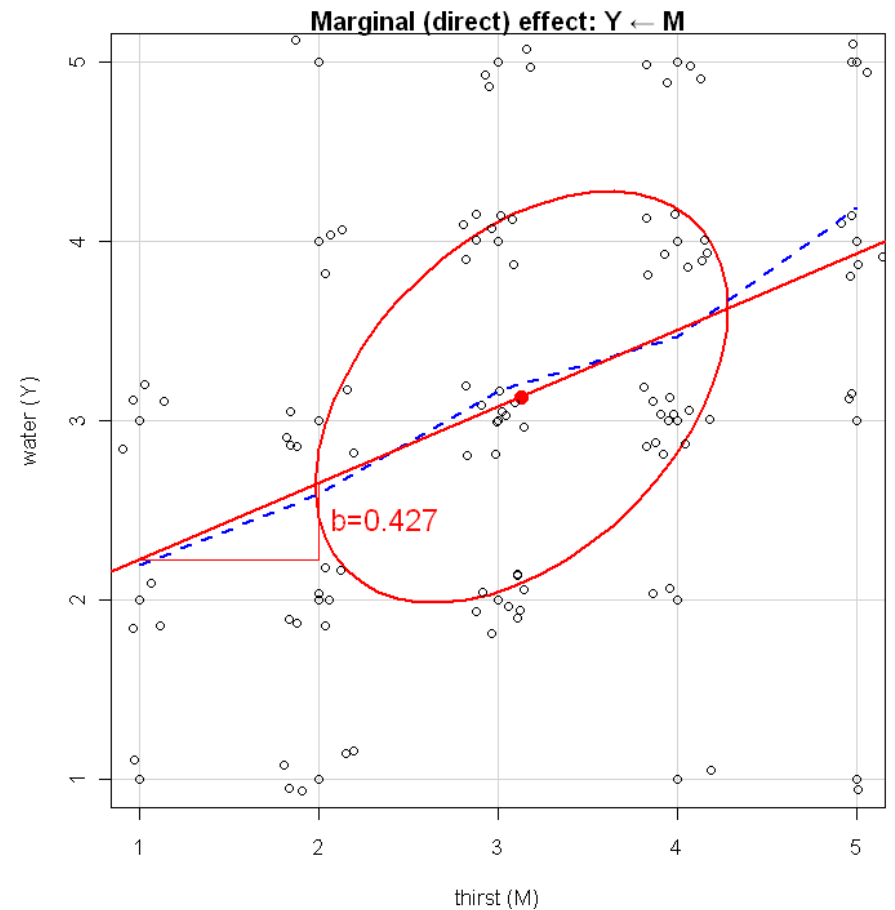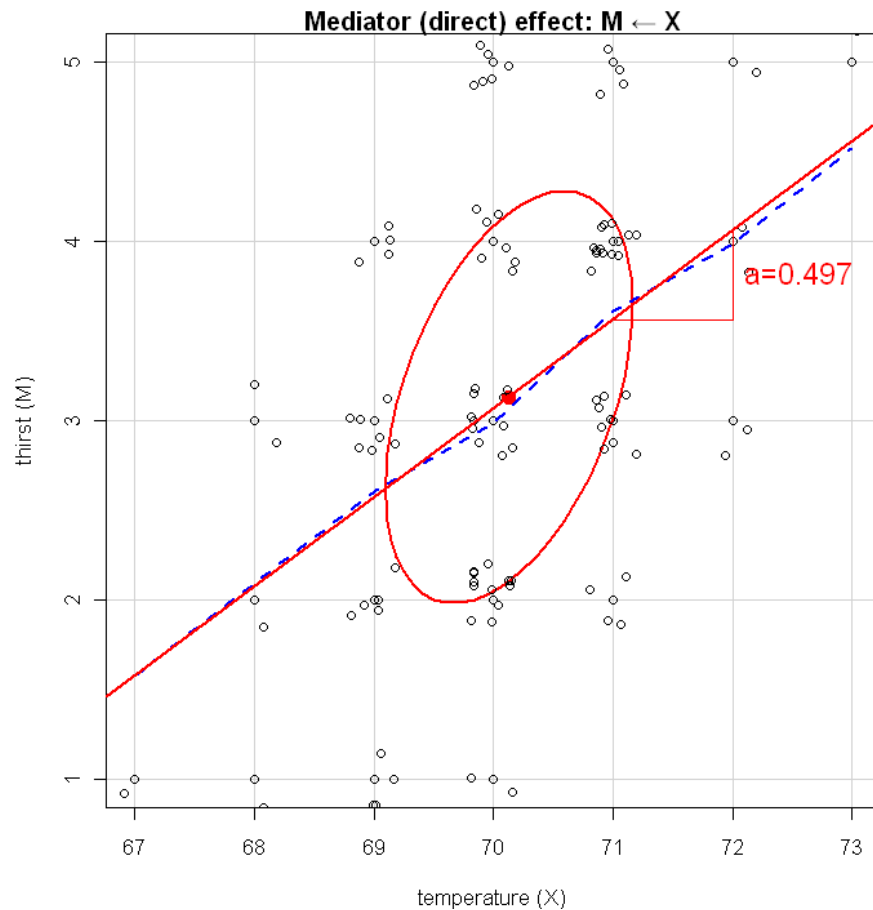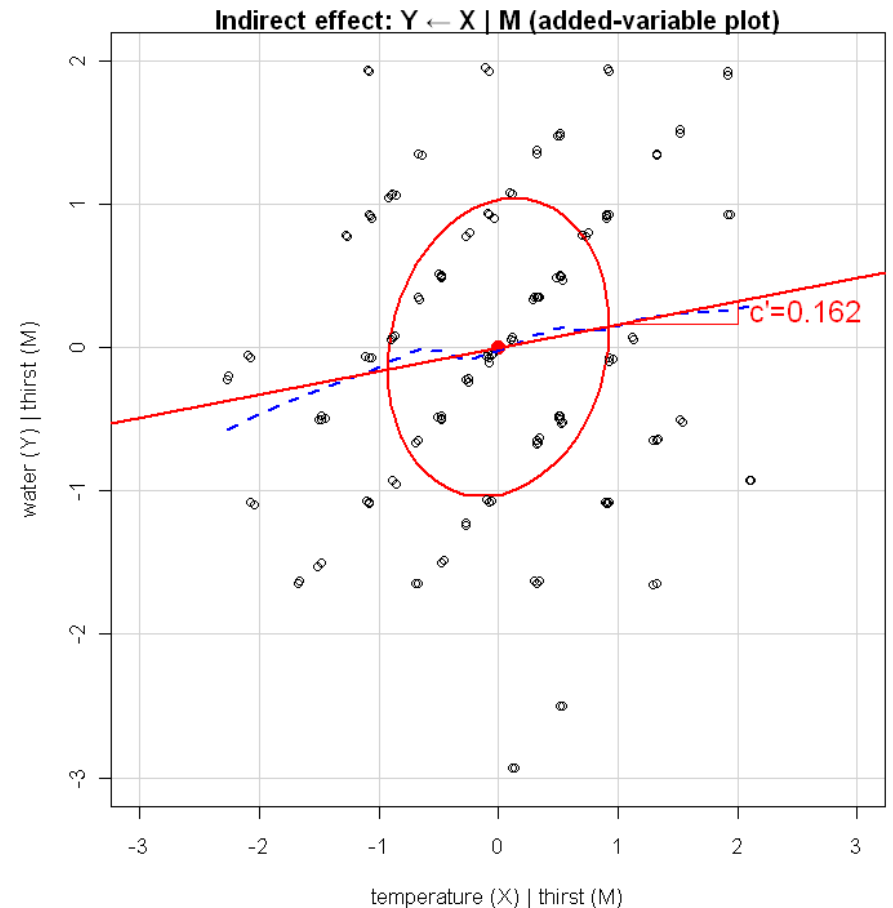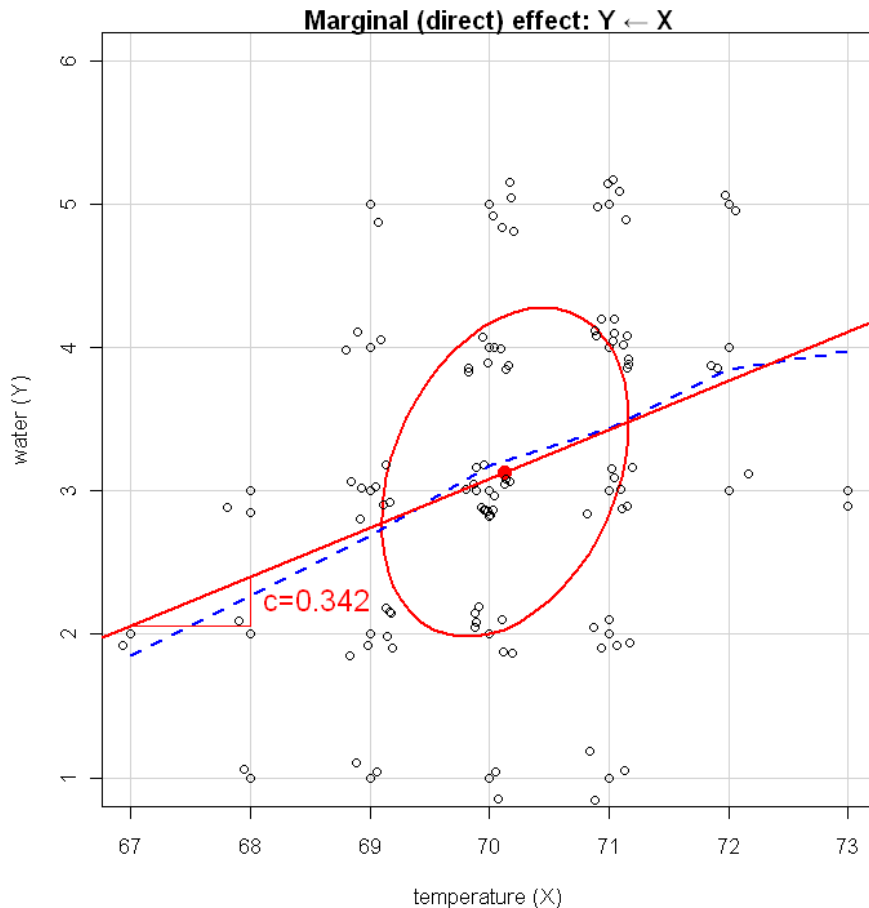
Direct effect (c): p<.05

Cond'l effect(c'): NS

Mediation: Is (c-c') ≈ 0 ?

These plots assure us that the simple relations of X →M → Y are reasonably linear, with no weird observations
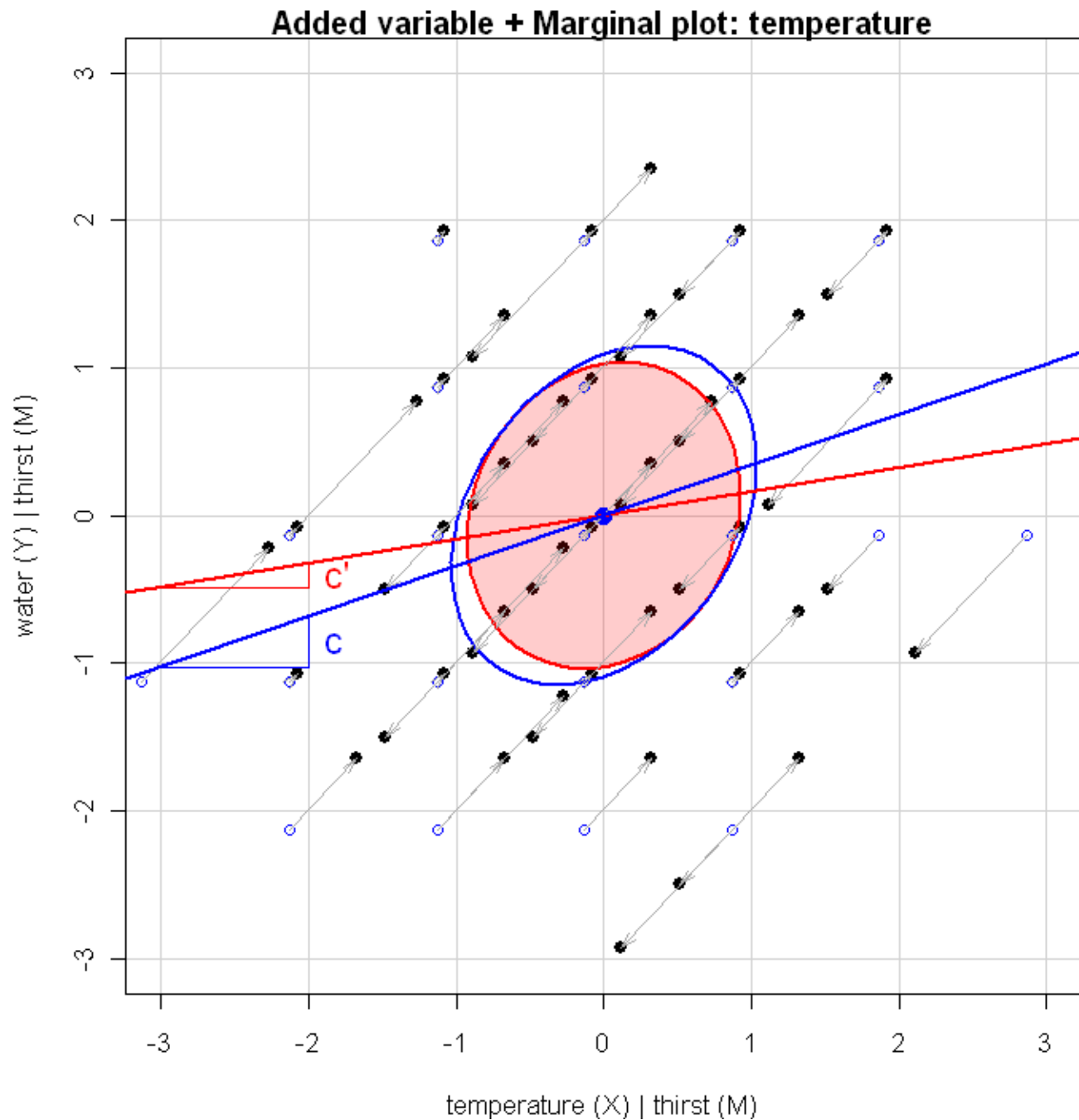
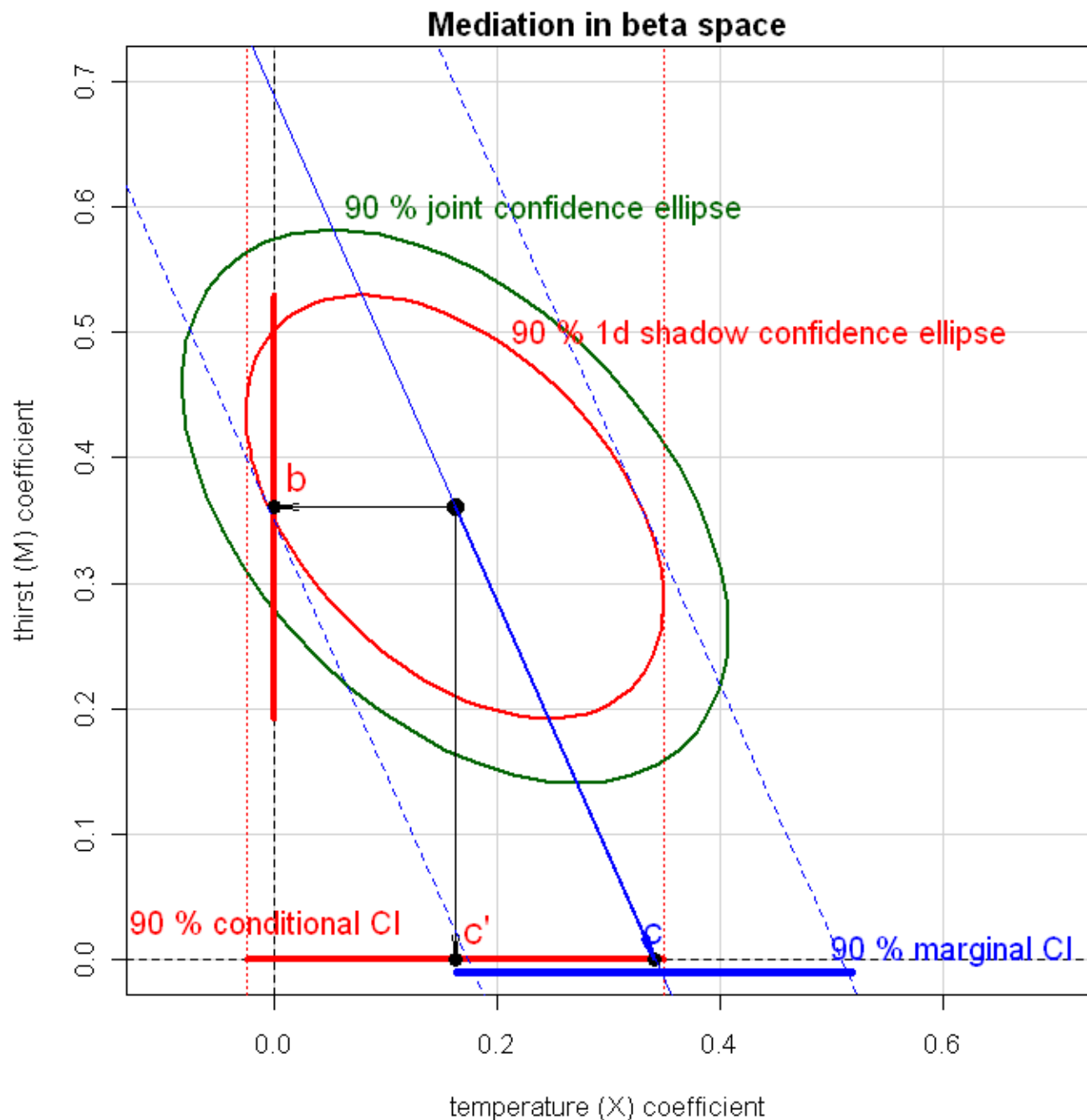The direct effect of X on Y can be seen in a standard scatterplot, enhanced with data ellipse and lowess smooth

The analog for indirect effects is the added variable plot. This works for any number of mediators.

The two plots can be overlaid by centering the data in the marginal plot

Arrows show the changes in (X,Y) points as M is controlled.

Added variable + Marginal plot: temperature

water (Y) | thirst (M)

temperature (X) | thirst (M)

For simple mediation,
all relations can be
seen in β space of
($β_X$, $β_M$)

Mediation in beta space

90 % joint confidence ellipse

90 % 1d shadow confidence ellipse

b

90 % conditional CI   c'   c   90 % marginal CI

thirst (M) coefficient

temperature (X) coefficient

Georges' sketch
(different notation)