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# The First (Known) Statistical Graph: Michael Florent van Langren and the "Secret" of Longitude

Michael FRIENDLY, Pedro VALERO-MORA, and Joaquín IBÁÑEZ ULARGUI

A 1644 diagram by Michael Florent van Langren, showing estimates of the difference in longitude between Toledo and Rome, is sometimes considered to be the first known instance of a graph of statistical data. Some recently discovered documents help to date the genesis of this graphic to before March 1628, and shed some light on why van Langren chose to display this information in this form. In the process, we discovered three earlier versions of the 1644 graph and one slightly later reproduction. This article describes these early attempts on the solution of "the problem of longitude" from the perspective of a history of data visualization.

KEY WORDS: Data visualization; History of statistics; Longitude; Michael Florent van Langren; Selenography; Statistical historiography; Thematic cartography; Uncertainty.

#### **1. INTRODUCTION**

Every picture tells a story (Rod Stewart)

In the history of statistical graphics (Friendly 2008), as in other artful sciences, there are a number of inventions and developments that can be considered "firsts" in these fields. The catalog of the Milestones Project in the history of data visualization (Friendly and Denis 2001) lists 70 events that can be considered to be the initial use or statement of an idea, method, or technique that is now commonplace. Some early examples related to the theme of this article are:

- map projections of a spherical earth and use of latitude and longitude to characterize position (Claudius Ptolemy, c. 150 AD)
- the first modern atlas, Teatrum Orbis Terrarum (Abraham Ortelius 1570)

• first world map showing lines of geomagnetism (isogons), used in work on finding longitude by means of magnetic variation (Guillaume le Nautonier 1604).

Among these, there is also:

• first visual representation of statistical data: variations in determination of longitude between Toledo and Rome (Michael Florent van Langren 1644). (Like others in his family and in this time, his name was written in various forms and in different languages: Miguel Florencio, Michale Florent, etc. We use the English version here, abbreviated as MFvL where it serves in the following.)

If this is truly the first exemplar of a graph of statistical data, van Langren should be canonized in this history, along with the contributions of William Playfair (1786, 1801) (invention of the line graph, bar chart, and pie chart), Charles Joseph Minard ("the best statistical graphic ever drawn"), Florence Nightingale (1857) (the use of the "coxcomb," or rose diagram for social and political advocacy), and others. Yet, van Langren and the history of his graph remain little known.

However, it is important to make clear (a) that many such 90 "firsts" are relative rather than absolute and (b) that "priority 91 chasing" for its own sake is often an unprofitable goal in his-92 toriography (May 1975). On the first point, many "first" devel-93 opments in the history of data visualization were preceded by 94 other contributions that count as a "first" under different specifi-95 cations. For example, the idea of a system of latitude and longi-96 tude for a map of the world was first proposed by Eratosthenes 97 in the third century BC; the idea of the "coxcomb"-a polar 98 area chart—goes back at least to André-Michel Guerry (1829), 99 and Nightingale almost certainly was introduced to this graphic form by William Farr (1852). On the second point, it is more useful to understand the context in which a significant historical event occurred.

This article describes the background and questions that led to van Langren's graph (Figure 1), conventionally dated to a 1644 publication, La Verdadera Longitud por Mar y Tierra (The true longitude for sea and land). We show why using a graph was effective for van Langren to achieve his communication goals. More importantly, some recently discovered letters by van Langren and others help tell the story behind this early graph and serve to date its genesis to early 1628.

#### 1.1 Other Early Graphs

We claim that this one-dimensional line graph by van Lan-114 gren is the first known visual representation of statistical data 115 (i.e., empirical data where uncertainty of the observations is 116

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Figure 1. van Langren's 1644 graph of determinations of the distance, in longitude, from Toledo to Rome. The correct distance is 16.5°. Source: Tufte (1997, p. 15).

14 nonnegligible) and so a brief justification of this claim is neces-15 sary.

16 First, we can set aside all purely cartographic representa-17 tions of geographical features (administrative boundaries, wa-18 terways, roads, cities, etc.) as outside the scope of the claim. 19 Thematic maps, that do attempt to show something more than 20 geographical features, did not arrive until the early 1700s 21 (Friendly and Palsky 2007).

22 The first known graph of any sort is an anonymous tenth-23 century conceptual depiction of cyclic movements of the seven 24 most prominent heavenly bodies through the constellations of 25 the zodiac described by Funkhouser (1936) and reproduced in 26 Tufte (1983, p. 28). Sometime around 1360, Nicole Oresme 27 [1323–1382] conceived of the idea to visualize the way two 28 physical quantities (e.g., time, velocity, distance traveled) vary 29 in a functional relation. In the Tractatus De Latitudinibus For-30 marum (Latitude of forms), published only much later (Oresme 31 1482), he used the terms "latitude" and "longitude" in much 32 the same way as we now use abscissa and ordinate, anticipat-33 ing Descartes (1637) in this regard by over 250 years. His dia-34 grams, shown in Figure 2, are the earliest abstract graphs we 35 know of, but they were not based on any data. Funkhouser 36 (1937, p. 277) said, "If a pioneering contemporary had pre-

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History Corner

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56 Figure 2. A page from Oresme's Tractatus De Latitudinibus For-57 *marum*, showing several graphical forms that might occur in a functional relation between physical variables. 58

sented Oresme with actual figures to work upon, we might have had statistical graphs 400 years before Playfair." But, empirical data that might have been displayed graphically were relatively unknown until around 1600.

It is recorded in the Milestones Project that around 1450, 76 Nicholas of Cusa (Nikolaus von Kues) [1401-1464] proposed 77 the idea of making a graph of the theoretical relation between 78 distance and speed, but no sources are known. Of particular im-79 portance in this time were the laws of motion that governed 80 falling bodies and projectiles. Albert of Saxony, Leonardo da 81 Vinci, and others proposed different laws relating these quan-82 tities. Shortly after 1600, Galileo began the experimental study 83 of the laws of relating time, distance, and speed of falling bod-84 ies, beginning with the (probably apocryphal) story of dropping 85 a cannonball and a feather from the Tower of Pisa. In 1604 86 he conducted experiments using balls rolling down inclined 87 planes, where he measured time with a water clock and sur-88 mised that the distance traveled was proportional to the square 89 of the elapsed time. 90

Figure 3 compares the proposals of Albert, Oresme, Leonardo, and Galileo in graphical form (Frautschi 1986), but none of these actually made a graph of data, as far as is known. See the works of Frautschi (1986) and Wallace (1968) for more on the early history of laws of motion and the development of the correct idea of uniform acceleration of falling bodies.

As far as we know, the next early data-based graph, after van Langren, was a 1669 graph (image: http://www.math. yorku.ca/SCS/Gallery/images/huygens-graph.gif) by Christiaan Huygens (Boyer 1947) of the distribution of life expectancy at any age, derived by curve fitting and graphical interpolation (smoothing) from John Graunt's (1662) Observations upon the Bills of Mortality. Yet even this is essentially the graph of a functional relation.

### **1.2 Family History**

The van Langren family is well known in the history of cartography. In the period from the mid 1500s through the end of the 1600s, three generations of this family achieved prominence as cartographers, globe makers, astronomers, and mathematicians, often with royal and state patronage. The genealogy of these individuals is shown in Figure 4. The brief description given here draws on the article by Keuning (1956) and the more recent and detailed studies in the book by van der Krogt (1993).

The van Langren family, starting with Michael's grandfa-115 ther, Jacob Floris van Langren (or Jacobus Florentius), born ca. 116

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Figure 3. Laws relating time, position, and speed of falling objects, as proposed by Albert of Saxony, Nicholas Oresme, Leonardo da Vinci, and Galileo. The points shown on the rightmost graph were used only to show the difference in functional form between da Vinci and Galileo. *Source:* Frautschi (1986, fig. 2.2).

1525, together with his son Arnold Floris, born ca. 1571, made
the first Dutch world globes, beginning around 1586 [dubbed
"LAN IA" by van der Krogt (1993)]. In 1589, they produced
a large version (52.5 cm diameter, "LAN II") that Jacob and
Arnold continued to enhance over the next 50 years.

The salient points of this family history—only as they relate to our present account—are as follows.

First, the van Langren globes were significant in the devel-24 opment of Dutch sea-faring trade that arose largely in conse-25 quence of the Dutch revolt against Spanish rule (over matters of 26 taxation, but principally of religion) that began under Philip II 27 of Spain in 1568. This conflict, called the Eighty Years War, 28 would last, on and off, for the next 80 years through the reigns 29 30 of Philip II to Philip IV of Spain until 1648. In 1595, the first 31 Dutch merchant fleet set out for the East Indies to circumvent the restrictions and blockades on the spice trade imposed by 32 33 Spain. By 1602, the Dutch East India Company (Vereenigde 34 *Oost-Indische Compagnie*) was established with a mandate to 35 carry out trade and colonial activities against Spanish inter-36 ests, and Dutch merchant ships would come to dominate world 37

trade in the seventeenth century. Numerous sources were used by these early Dutch navigators, but the van Langren globes were particularly prized. In September 1592, Jacob Floris van Langren was granted a charter (monopoly) for 10 years on the production of globes in the Low Countries by the States General.

Second, the van Langren family enterprise of globe and map making, together with the copper plates and engraving tools, passed to his sons, Arnold Floris and Hendrick Floris van Langren, when the elder Jacob decided to retire at age 75 (van der Krogt 1993, p. 128). For navigational purposes, globes were often produced in pairs—a terrestrial globe, showing a map of the earth, and a celestial globe, showing a map of the heavens. But the initial van Langren globes of 1586 and 1589 had no celestial counterpart, so, in April 1590, Arnold was sent to visit Tycho Brahe on the island of Hven to make copies of Brahe's star catalog for the production of a celestial globe. The visit was evidently not particularly successful, and Hendrick was sent to complete the task in April 1593. A celestial companion to the 1586 globe (LAN IB) was produced in 1594; a mate to the larger 1589 globe did not appear until 1630.

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40	Egbert Floris	Jacob Floris var	n Langren		98
41	van der Erve	globemal	ę		99
42		died in Alkmaar, 1610			100
43	died 1588		ar, 1010		101
44					102
45			-		103
46	Arnold Floris	Hendrik Floris	Annetge	Fijtge	104
47	van Langren	van Langren	van Langren	van Langren	105
48	engraver and	engraver	ca. 1575	married	106
49	globemaker	ca. 1574-1648	married 1596	Jochum Willems	107
50	ca. 1571-1644		Jacob Reyers	cheeseseller	108
51	1		•	at Alkmaar	109
52					110
53	(not inclusive)				111
54	Michael Florent				112
55	van Langren				113
56	1598-1675				114
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58	Figure 4.	van Langren family genealogy.	Source: van der Krogt (1993	3, p. 91).	116

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1 Finally, by all accounts, Arnold was a skilled engraver, but he 2 was not highly possessed of financial and business acumen. By 3 late 1607 or early 1608, he had accumulated sufficient debts that he was forced to flee from Amsterdam to the Spanish-4 5 ruled southern provinces-in such haste to escape the bailiff 6 that he left behind his household goods and engraving tools. To 7 re-establish himself, he offered a large terrestrial globe to the city of Antwerp. We surmise that his skills also included the 8 9 social abilities to attract supporters in high places. In September 1609, he succeeded in being named official Spherographer 10 to Archduke Albert of Austria and his wife, Infanta (Princess) 11 Isabella Clara Eugenia of Spain (daughter of Philip II), rulers 12 of the Spanish Netherlands. 13

After Albert died in 1621, Isabella was appointed governor 14 on behalf of Philip IV and continued to support Arnold's work 15 on terrestrial and celestial globes and the accompanying man-16 uals that were needed for mariners at sea. In all probability 17 some of this additional work was carried out by Arnold's son, 18 Michael Florent van Langren, the subject of this essay. 19

#### 1.3 The Problem of Longitude

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22 In relation to the history of data visualization, the most im-23 portant scientific questions of the sixteenth and seventeenth 24 centuries were those concerned with physical measurement-of 25 time, distance, and spatial location. These arose in astronomy, 26 surveying, and cartography and related to practical concerns of 27 navigation at sea, exploration, and the quest for territorial and 28 trade expansions among European states. Here, we describe one 29 of these-the problem of longitude-but only so far as it re-30 lates to, and provides context for, van Langren's graph. [See the 31 work by Andrewes (1996) for a variety of perspectives (mathe-32 matics, cartography, navigation, etc.) on the history of the lon-33 gitude problem and attempted solutions. Sobel (1996) provided 34 a popular account (deprecated by historians) of the first accu-35 rate solution, the invention of the marine chronometer by John 36 Harrison in 1765.]

37 Among the most vexing—and also consequential—problems 38 was that of the accurate determination of longitude, both on 39 land and at sea. Latitude, a N-S position relative to the equator, 40 has a physical 0 point on the terrestrial sphere and a conven-41 tional range to  $\pm 90^{\circ}$  at the poles; this could easily be found with 42 a sextant or other device for measuring the angle (declination) 43 of the sun, moon, or given stars, using tables of these positions 44 that had been common for many centuries. Longitude, the E-W 45 position, has no natural 0 point, and no natural points of refer-46 ence; the only physical fact is that the scale of  $0^{\circ}$ -360° around 47 any parallel corresponds to the 24-hr rotation of the earth, or 48 15° per hour. Errors in navigation led at best to much longer 49 travel times and short rations or starvation for seamen, at worst 50 to numerous shipwrecks and maritime disasters.

51 For the present purposes, it is both convenient and histori-52 cally appropriate to illustrate this with two diagrams van Lan-53 gren used to explain the problem of longitude to the council 54 of King Philip IV of Spain in 1632; the context in which this 55 occurred is described in Section 2.2. In Figure 5 he says, in 56 essence: consider trying to determine the location on earth of 57 point D in relation to point B (say, Toledo), either on land or 58 at sea. To determine the distance in longitude, you must refer



Figure 5. van Langren's 1632 first diagram, to explain the problem of determining longitude on land and at sea. Source: van Langren (1632).

point D to its meridian that passes through point E at the equator and refer point B to its meridian passing through point C on the equator. Then, you could calculate the distance between points B and D on land or at sea.

But longitude at sea is a far more difficult and consequential problem. In Figure 6, van Langren shows the path of a ship that has left a port at F (with longitude at the equator, H) and has arrived at point L (longitude I), by traveling along the curved path shown from F to L. How can a navigator know how to return to port (F) or to sail further west to an island at point M?

What was needed for longitude was an accurate means to 90 determine the difference in time between where you are and 91 some fixed reference point. For example, if a navigator knew 92 the time in Toledo at the same instant the sun reached its zenith (local noon), the difference between local noon and Toledo time would give the longitude difference, and similarly for any other celestial event. But, how could one know the time in Toledo 96 when located at point D?

In the absence of accurate clocks, the problem was difficult 98 on land, but even more so at sea, where celestial observations 99 were subject to constant and changing motion and therefore 100 more prone to error. As early as 1530, Reginer Gemma-Frisius 101 [1508–1555] proposed a theoretical description—eventually 102 proved to be correct-of how to determine longitude by a dif-103 ference between clocks; but pendulum and other mechanical 104 clocks (Galileo had used water clocks for his experiments in 105 motion; Huygens obtained a patent for the pendulum clock 106 in 1657) would remain insufficiently accurate for at least 100 107 years on land, and over 200 years for longitude at sea. 108

What else could serve as substitute for a reliable chronome-109 ter? Heavens above! In 1514, Johannes Werner [1468–1522] 110 published a translation of Ptolemy's Geographia. In a commen-111 tary on Book I: Chapter IV, he proposed what became known 112 as the lunar distance method, that is, determining longitude by 113 measuring the angle between the moon and some star or the sun, 114 with the use of a nautical almanac recording those positions at 115 given dates and times at a fixed position. 116

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Figure 6. van Langren's second 1632 diagram, to explain the more specific problem of determining longitude at sea. *Source:* van Langren (1632).

One reason for this great attention to the longitude problem was a series of substantial prizes offered by various European countries for "the discoverer of longitude." The first was offered by the Spanish King Philip II in 1567. After Philip III came to the throne in 1598, a prize of 6000 ducats, plus a pension of 2000 ducats for life was offered. Other rewards were offered by Holland and France, culminating in The Longitude Prize, be-gun in 1714 by the British government and resulting in awards totaling over £100,000 (O'Connor and Robertson 1997). [The proximal stimulus was one of the worst ever maritime disasters on 22 October 1707, in which five ships commanded by Admi-ral Cloudsley Shovel struck the Isles of Scilly, resulting in 2000 sailors killed. Various specific prizes were offered: £10,000 for a method accurate to within 1° of longitude (111 km, or 60 nautical miles) to £20,000 for accuracy within 0.5° (56 km, 30 nautical miles), plus other awards for contributions thereto.] 

Over the next 200 years, problems that stemmed from at-tempts to solve the longitude problem by means of celestial observation and reference tables would occupy the attention of the best astronomers and mathematicians of the times. For exam-ple, in 1612 (shortly after the invention of the telescope) Galileo tabulated the orbits of the four largest moons of Jupiter and pro-posed that sufficiently accurate knowledge of their orbital peri-ods could serve as a more precise celestial clock. Attempts to increase the precision of the lunar distance method were ad-vanced by Edmund Halley around 1683 (times of occultations of a star by the moon). 

After the publication of Newton's universal theory of gravita-tion in 1687 (*Principia mathematica*), the mathematical theory of planetary and lunar motion became more advanced. Tobias Mayer (1750), using maps of lunar features first developed by van Langren, made observations sufficiently precise to allow a mathematical and statistical account of the small perturbations ("librations") of lunar motion. This would serve as an early con-tribution to the theory of least squares as a method of combining observations subject to error (Stigler 1986, pp. 16-24).

This was the ring into which Michael van Langren threw his
hat. A large part of his lifetime would be occupied by the problem of determining longitude. In this endeavor he was not particularly successful in providing a solution. Yet, along the way,
he made a number of important contributions that we describe
below, with a focus on his statistical graph and its wider context.

## 2. VAN LANGREN'S GRAPH

In this section we first consider the 1644 graph (Figure 1) from *La Verdadera* in relation to van Langren's presumed presentation goals and why this simple graph should be celebrated for graphical excellence, beyond just being a 'first.' We then describe the discovery of earlier letters by van Langren that (a) serve to date the origin of this graph to sometime before March 1628 and (b) provide a detailed history of the context and motivation that led him to adopt this graphic form.

#### 2.1 Why the First Graph Got It Right

Van Langren's 1644 graph (Figure 1) was designed to contribute to a more precise determination of longitude, particularly for navigation at sea as noted above. There is, of course, a single, correct value for the distance in longitude between Toledo and Rome, but that value, 16.5°, would remain imprecisely known for over 100 years.

Looking at this graph, what can we see? On the graph, he shows 12 estimates of the longitude distance, on a scale of  $0^{\circ}$ –  $30^{\circ}$ , each with a label (written vertically) for the name of the person who made that determination. This graph is remarkable in the history of data visualization for several reasons.

First, it would have been easiest for van Langren to present this information to the Spanish court in tabular form, showing (Name, Year, Longitude, Where), as suggested in Figure 7. Such a table could have been arranged to highlight authority (sorted by name), or priority (sorted by year), or the range of values (sorted by longitude value).

However, only a graph speaks directly to the eyes and shows the wide variation in the estimates; note that the range of values covers nearly half the length of the scale. It also appears from the graph that van Langren took as his overall summary the center of the range, where there happened to be a large enough gap for him to inscribe "ROMA."

Second, van Langren's graph serves as a compelling visual example of the idea of bias in statistical estimation. In Figure 1 a small arrow marks the true longitude distance (16.5°). This aspect can be seen more clearly by overlaying van Langren's

# Sorted by Longitude

	Sorted by Long	itude		Sorted by Priority
Longitude	Name	Year	Where	Year Name Longitude When
17.7	G. lansonius	1605	Flanders	150 Ptolomeus, C. 27.7 Egy
19.6	G. Mercator	1567	Flanders	1463 Regiomontanus, 25.4 German
20.8	I. Schonerus	1536	Germanv	1530 Lantsbergius, P. 21.1 Flander
21.1	P. Lantsbergius	1530	Flanders	1536 Schonerus, I. 20.8 German
21.5	T. Brahe	1578	Denmark	1542 Orontius 26.0 France
25.4	I. Regiomontanus	1463	Germany	1567 Mercator, G: 19.6 Flander
26.0	Orontius	1542	France	1567 Clavius, C. 26.5 German
26.5	C. Clavius	1567	Germany	1578 Brahe, T. 21.5 Denmai
27.7	C. Ptolomeus	150	Egypt	1582 Maginus, A. 29.8 Ita
28.0	A. Argelius	1610	Italy	1601 Organus, D. 30.1 German
29.8	A. Maginus	1582	Italy	1605 Iansonius, G. 17.7 Flander
30.1	D. Organus	1601	Germany	1610 Argelius, A. 28.0 Ita

# Figure 7. Two of the possible tables van Langren might have used. Left: sorted by longitude, to show the range. Right: sorted by year to show priority or trend. The Year given in these tables is approximate, as van Langren did not cite his sources by date.

graph on a modern map as shown in Figure 8: the previous estimates of longitude distance were extremely far from being correct and place Rome anywhere from the Adriatic Sea to Greece 

Finally, van Langren's graph is also a milestone as the earli-est known exemplar of the principle of "effect ordering for data display" (Friendly and Kwan 2003): graphs and tables are most effective when the information is arranged to highlight the fea-tures to be seen. In this case, it is clear that van Langren's main presentation goal was to show the enormous range of differ-ences among the greatest known astronomers and geographers. As such, the graph is all the more remarkable for its focus on uncertainty or variability of observations, a topic that did not re-ceive serious attention until roughly 100 years later (see Stigler 1986). Thus, we conclude that the first graph did indeed 'get it right.'

# 2.2 The 1628 Letters: Stake a Claim

Michael van Langren, like other scholars of his time, was a prodigious writer of letters. (For example, Quetelet 1864, pp. 247-248, described collections of letters to and from van Langren in several archives. Other collections of his letters were described or contained in Moreau 1957; de Vyver 1977.) Unlike those with independent means of support, and without a university education, van Langren had to depend on patronage to sustain his work and earn a livelihood. Most likely through

his family connection (Arnold's son), in 1626, at age 28, he obtained a commission to make a map of the new canal constructed between the Meuse in Belgium and the Rhine in Germany. This map, titled Fossae S. Mariae descriptio... (Keuning 1956, fig. 1), was dedicated to Isabella Clara Eugenia, governor of the Netherlands, who would soon become his patron. Around this time, with the intercession of Isabel, he received an appointment as Royal Cosmographer and Mathematician to King Philip IV, for which he would come to receive an annual retainer of 1200 écus, apparently a considerable sum. Michael van Langren had arrived: financially, and in the Spanish court.

Recently, the third author found in the archive of his family some documents related to fray Íñigo of Brizuela, President of the Counsel of Flanders until 1629. These contain (a) an undated letter by van Langren to Isabella. In this, he requests that Isabella intercede on his behalf to King Philip to grant him a concession to make more accurate maps for navigation; (b) an undated letter from Isabella to King Philip giving reasons to support this concession; (c) a letter in French, dated March 9, 1628 by Dr. Jean Jacques Chifflet, in service to Isabella, written to fray Íñigo of Brizuela in support of van Langren's request.

The letter by van Langren contained the graph and table shown in Figure 9. Because of the sequence of the three letters, it is possible to date this letter to sometime before March 1628, possibly in the early part of this year. In the letter (below), he describes himself as "Mathematician to His Majesty," so this





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exemple siguinto. Las distancias de Mercato oannes Sconerus 2 Roma y Foledo segun 3 los autores. Leg. de Alle El Globo \_\_ 195 Gerard 4 Escala de 100 leguas de Allemaña para -195 conocer las differencias del un autor a la otra 5 Toledo. Gerd. Mercat \_ 210 6 Ivan sconer\_ 230 7 Orontius - 269 8 15 Joan Regiot\_283 9 grados de la Longitua P. Claucus - 294 10 Ptolomeus - 307

Figure 9. The graph and table of determinations of longitude, from the letter of van Langren to Isabella Clara Eugenia, ~1628. Source: Archive of the counts of Castilfalé, Burgos, Spain.

could not date before 1626. Thus, van Langren's graph had its origin at least 16 years before the 1644 publication of La Verdadera that contained Figure 1.

Van Langren's letter to Isabella was transcribed from old to modern Spanish and then translated to English. It is worth printing the English translation to see the context and motivation for the graph. (Available online at http://www.math.yorku.ca/SCS/ Gallery/langren/VanLangren1628.pdf.)

#### Most Serene Highness

26 Miguel Florencio van Langren, Mathematician to His Majesty, says that his Grandfather and his Father, Cosmographer to His 27 Majesty, have been the first who have invented Globes for the 28 direction of navigators, and the supplicant, emulating them has 29 attained with great study and attention some fundamental and 30 concealed aspects of the aforementioned art as well as others, 31 and one of the main is that of Longitude, by which it is possible to lay out perfectly all the Terrestrial description, which 32 has countless errors as can be seen in the writings of different 33 authors, because comparing two maps or tables of longitude of 34 different authors, by no means do they concur between them as 35 Your Serene Highness will see in the following example. 36 If the Longitude between Toledo and Rome is not known with certainty, consider Your Highness, what it will be for the West-37 ern and Oriental Indies, that in comparison the former distance 38 is almost nothing. So to amend these deficiencies and to find 39 the true distances of the towns of the Earth, it would be neces-40 sary that Your Serene Highness be pleased to supplicate to His 41 Majesty (as in the example of the Queen Isabel of Castile) to dis-42 patch a Patent so that he can send his corresponding and printed instructions for all the Earth, both to the East and to the West, 43 ordering in it that all interested in the art observe what the sup-44 plicant advises them, promising that many benefits will derive 45 for navigation, and eternal memory for His Majesty and Your 46 Highness, for having ordered this general correspondence of the art, and Your Most Serene Highness will receive it very particu-47 larly. 48

Thus, he makes explicit that the purpose of drawing a graph is to show the "countless errors" in the determination of longitude distance between two relatively well-known locations. In its intent, the letter can be read as a classic example of the syllogism of grantsmanship as a patronage request:

- 1. Longitude distance between Toledo and Rome is subject to 55 56 large errors.
- 2. By extension, the errors in longitude between Toledo and the 57 58 Western and Oriental Indies must be far greater.

3. Therefore, grant me a Patent to sort this out, promising many benefits for navigation.

Viewed as a grant application, what is missing from this letter is a Method or Proposal paragraph indicating how he intends to solve the longitude problem. This would be a goal that occupied his attention over the rest of his life. Yet, perhaps fearing that others would steal his thunder or claim priority for his method, he intentionally kept the details secret. In La Verdadera (p. 8), Langren included a description of a new method to determine longitude, but in the form of a cipher whose meaning he said he will reveal to the king, presumably after suitable compensation. He claimed that this was communicated to the king via Isabel in 1625.

Some differences between this early graph (Figure 9) and the 1644 version (Figure 1) are worth noting:

• The most important difference is that in Figure 9, he includes a separate scale showing longitude distances in German leagues (Gl.), with a corresponding table of those values in the margin. The values in Gl. can also be seen faintly beneath the horizontal axis, as if he first recorded these values along the scale and then erased them. The use of double scales is common today (e.g., showing temperature in °F and °C), but it is remarkable that such annotations would appear on the first statistical graph.

99 Why did he do this? First, it is reasonable to assume that 100 he felt the need to show that the values on the graph were ac-101 tually based on real data. A second inference is that the values 102 shown were originally expressed in German leagues and then 103 translated to degrees longitude. (A graphical conversion from 104 his scales shows that 100 Gl. =  $8.9375^{\circ}$ , or 111.88 Gl. = 105 10°. Translation to modern units is more difficult to infer. As we note below, at the equator,  $1^{\circ}$  in longitude = 111 km, 106 107 so, approximately, 1 Gl. = 10 km. However, at the latitude 108 of Toledo,  $1^{\circ}$  in longitude = 85.3 km, so, approximately, 109 1 Gl. = 7.62 km.

The graph and table in Figure 9 show only 7 values instead 110 111 of the 12 shown in Figure 1. Among these, Figure 9 contains a value 17.7° attributed to the "globe of Langren." Among all 112 113 the images of van Langren's graph we have found, this is the only occurrence of an attribution that appeared just once. Yet, 114 this value was off by only 7.2% in relation to all the other au-115 thorities named in all his graphs. (Among the five versions of 116

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van Langren's graph we have found, this is the value with the second smallest positive bias, after those recorded for Iansonius (Jan Jansson [1588-1664]), for whom the average bias was 1.026°, or 6.2%. The particular van Langren globe referred to here is unknown. A close reading of van der Krogt 6 (1993, chaps. 3 and 7) suggests the guess that this may be one of the large terrestrial globes ("LAN II") produced by Jacob 8 and Arnold van Langren beginning in 1598 and modified in other copies over the next 50 years.)

Moreover, the value ascribed to "Orontius" (Oronce Finé) 10 here is located at 24° of longitude, rather than the value 26° 11 12 in the 1644 version. We analyze differences among these and other versions in more detail in the online Appendix. 13

14 • Finally, the word ROMA is not inscribed in the convenient gap along the horizontal axis as in Figure 1, but it is written 15 vertically in front of the name of each authority. Thus, the 16 inference that the word ROMA in Figure 1 marked a central 17 18 (typical, average, or best estimate) value for the true longitude distance is incorrect. (As far as is known, the idea of 19 20 combining observations by averaging was first expressed in 1635 by the English astronomer Henry Gellibrand [1597-21 1636] (Gellibrand 1635). Later developments of this idea oc-22 curred only in the mid to late 1700s (Thomas Simpson, To-23 bias Mayer, Pierre Simon Laplace), finally arriving at the idea 24 of the mean as a least squares estimate around 1800 (Adrien 25 26 Legendre, Carl Friedrich Gauss).)

# 2.3 Letters of 1632–1633: Eyes on the Prize

We are not quite done with the analysis of van Langren's early letters related to this graph of longitude. In the course of research for this article, another similar collection of letters (van Langren 1632) was found in the Archivo General de Indias (General Archives of the Indies) in Seville, and cataloged as Estudios Sobre la Longitud, Patronato, 262, R.7. There are five letters, including two by van Langren describing his ideas for the determination of longitude, (these may be accessed online through the Portal de Archivos Españoles, http://pares.mcu.es, searching for "van Langren"), with supporting letters by a Juan Osvaldo de Brito. The first of these, by de Brito, is dated February 28, 1632 and is essentially a cover-memo to the War Subcommittee of the Council to Philip IV, concerning the attached proposal by MFvL that had been presented to the Council January 7 of that year.

Both letters by van Langren contain slightly different ver-59 sions of the graph of longitude distances. Figure 10 shows the 60 version contained in the 1632 letter. Here, for the first time we 61 know of, he attempts to deal with the general problem of map-62 ping a spherical earth in terms of latitude and longitude, de-63 scribed earlier (see Figure 5). Consequently, in the 1632 version 64 of the graph, he explicitly represented the estimates for Rome 65 along a separate line for the parallel of Rome. 66

In addition, he showed 13 separate points on the graph, but a 67 close inspection shows that two of the points each cite two au-68 thorities ([Ptolomeus, Orontius]; [Leonitrus, Origanus]), for a 69 total of 15 citations. These include five sources [Algunas ("Al-70 gunas" here refers to "various" sources that he could not or did 71 not want to disclose) maps, Bertius, Algunas map & globe, P. 72 Apianus, Leonitrus] not seen in any other versions of the graph. 73 The letter of 1633 uses the same data, but omits the separate 74 parallels for Toledo and Rome. 75

Beyond the details of differences among these graphs, the 76 texts of these letters give further insight into van Langren's mo-77 tivation for writing them. He begins with the statement that 78 MFvL, mathematician to His Majesty, has uncovered some 79 "very important secrets" with regard to the computation of lon-80 gitude, on land and at sea. There follows a presentation of the 81 general longitude problem (Figure 5) and the additional prob-82 lems for longitude at sea (Figure 6). In this, he illustrates the 83 habitual errors of navigators that result from inaccurate maps 84 and charts, using his current version of the graph of longitude 85 determinations, now with additional sources (Figure 10), and 86 reminds His Majesty that he has been granted a royal mandate 87 to carry this out. 88

The final paragraphs get to the main points:

And in this regard His Majesty offered to the Inventor of such solution great rewards, and in particular to Luis Fonseca 6000 ducats every year, and then to Juan Arias, 2000 ducats every year for a lifetime; so if his Majesty sends to this Supplicant the assurance of a prize that his Royal Highness judges appropriate, he (the Supplicant) will report the aforementioned secret to His Majesty, because finding this invention and not getting any reward would be honorless. (My person) also supplicates that His Majesty shelters him against the objections that some could put, saying that my invention is old and known and that I should not interfere in this, as I think that it must be sufficient if it is good and useful.

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Evidently, after these two letters for support, he was successful on both accounts, without even revealing the "secret." He



Figure 10. van Langren's 1632 graph of determinations of longitude. Here, he cites additional sources and makes clear that Toledo and Rome are located on slightly different parallels of latitude. Source: van Langren (1632).

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received some unknown but handsome compensation, and, in the following year (van Langren 1634), he felt free to announce "to all the professors and lovers of mathematics, on the proposition of longitude, by sea and land": it has been done by MFvL, mathematician to his Catholic Majesty.

3. VAN LANGREN'S DATA

At least four questions naturally arise from the above discussion: (a) What were the sources of the estimates of longitude that van Langren displayed in these graphs? (b) Why did he choose Toledo and Rome? (c) How did the authorities and values displayed differ across the versions of the graph? (d) Why were the estimates so biased upward? These questions take us into the realm of statistical historiography (Friendly 2005), that is, the use of statistical thought in the analysis of historical sources. For reasons of length, these topics are discussed in the online Appendix included in our supplementary materials.

# 4. OTHER WORK: LONGITUDE AND SELENOGRAPHY

23 Although van Langren's cipher remains undecoded, the "se-24 cret" that van Langren alluded to in his 1632/33 entreaties to 25 the Spanish court is related to a possible improvement in the lu-26 nar method that he surmised from telescopic observations. This 27 would turn out to be his principal contribution to astronomy, 28 wherein he is better remembered than in the history of statisti-29 cal graphics.

30 As early as 1628, he conceived the idea of using the rota-31 tion of the moon-rather than its mere position in the sky-as a 32 more accurate celestial clock. By timing the occurrence of sun-33 rise or sunset on identifiable lunar peaks and craters, one would 34 have a nearly continuous set of reference events with which lo-35 cal time could be accurately determined.

36 Two things were needed to make this idea practical for the 37 determination of longitude at sea. First, it required an accurate lunar globe or set of maps that named the peaks, craters, and 38 39 other lunar features so that they could be easily recognized. Sec-40 ond, it required a set of ephemeris tables, recording the onset 41 in standard time of sunrise (lightening) and sunset (darkening) 42 events on the days of the lunar cycle.

43 Van Langren spent a few years in the early 1630s at the court 44 in Madrid, during which he attempted to enlist support for this 45 project and made plans for the preparation of a collection of lu-46 nar maps and diagrams, together with a "user guide" containing 47 instructions for the calculation of longitude from observations 48 of the lunar features he would catalog. Because he would be the 49 first to comprehensively map the lunar features, he proposed 50 to have "the names of illustrious men applied to the luminous 51 and resplendent mountains and islands of the lunar globe," a 52 prospect that evidently pleased King Philip.

53 In a letter dated May 27, 1633 (de Vyver 1977; Whitaker 54 2004), Philip encouraged Princess Isabella (van Langren's pa-55 tron) to fund this project. Isabella's death in December of this 56 year delayed both his source of funds and enthusiasm for timely 57 completion of this work. Over the next decade he produced 30 sketches of lunar features in different phases of its cycle. 58

EL FLORENTIVS VAN LANGREN Figure 11. van Langren's 1645 map of the moon, Plenilunii Lumina Austriaca Philippica, dedicated to King Philip and showing 325 topographic names he had assigned to lunar features. Source:

News that others (Johannes Hevelius and P. Juan Carmanuel y Lobkowitz) were planning their own projects in mapping the lunar surface propelled him to complete what is now regarded as the first comprehensive lunar map.

http://www.lpod.org/?m=20060128.

In early 1645 he produced a manuscript version (Whitaker 96 2004, fig. 25) showing 48 named topographic features. Correspondence among van Langren, his friend E. Puteanus, and oth-98 ers in the Spanish court (Bosmans 1903) date this to early February of this year. The final version, engraved in copperplate in March 1645 (Figure 11) identifies 325 named locations. [Of the manuscript version, only three copies were known to exist by Bosmans (1903, p. 110); Whitaker (2004, p. 40) could locate only four extant copies of the engraved version.] Having exhausted his list of "illustrious men," and even adding several saints, he still had more features than names. Beyond the titular dedication to King Philip (Plenilunii ... Philippica), the Infanta Isabella Clara Eugenia appeared three times in his nomenclature, in recognition of years of patronage. See the book by Whitaker (2004) for a comprehensive account of the naming of lunar features.

Van Langren never completed the manual and tables describ-112 ing exactly how his lunar map could be used. Moreover, al-113 though his scheme for longitude determination based on a de-114 tailed lunar map did offer the opportunity for greater precision 115 than previous lunar methods, the relatively slow speed with 116



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which the lunar peaks became illuminated or vanished set hard

limits on the precision this method could achieve. Nevertheless, he was the first to produce a comprehensive lunar map,
and his own toponym for the crater *Langrenus* survives to this
day, along with about half of his other names.

Michael Florent van Langren died in Brussels in May 1675 long before his ideas for the determination of longitude could be improved upon by others. Variations of the lunar method advanced considerably over the next 100+ years, providing greatly improved accuracy based on more detailed and precise observations and mathematical theory to account for the libra-tions of the moon. As noted earlier, Tobias Mayer (1750) made significant contributions in this regard. In this, he used a luna map of greater precision (Whitaker 2004, fig. 52), the first to be based on accurately measured positions on the lunar surface His tables and calculations offered the prospect of accuracy to within the 1° (111 km) range for at least the smallest prize des-ignated by the British Board of Longitude. Mayer died in 1762 but his work would ultimately earn his widow a £3000 prize in 1765, and smaller prizes of £300 were awarded to both Got tfried Leibnitz and Leonhard Euler as contributions to Mayer' work. 

#### 5. CONCLUSIONS

As in the opening quotation, "Every picture tells a story," the aim of this article has been to uncover the larger history behind van Langren's 1644 graph (Figure 1), often considered the first exemplar of a graph of statistical data. From the discovery of his early letters, we have learned that the origin of this first one-dimensional graph goes back at least 16 years earlier, to the letter dated before March 1628 to Isabella Clara Eugenia (Figure 9). Moreover, the sequence of development of the earlier versions of this graph helps to shed some light on aspects of the determination of longitude in his time.

We have shown that, at the time of the earliest (1628) version, he already had in mind a possible and more accurate solution to the longitude problem, based on lunar rotation and the lunar maps he had begun to contemplate. He did not complete the lunar map until 1645, and never finished the remaining parts (tables and instructions for their use) to make this a practical method. Consequently, he is better remembered in historical studies for his lunar maps than for his contributions to the longitude problem in his time.

Nevertheless, we have also shown that Michael Florent van Langren should be remembered as well for his contributions to statistical graphics and data visualization. His one-dimensiona graphs, showing the wide variability in the previous determi-nations of longitude distances between Toledo and Rome, are outstanding to this day as examples of clear visual presentation from a time before the ideas of uncertainty of empirical esti-mates or even of representation of empirical data values along an axis were known. 

#### SUPPLEMENTAL MATERIALS

Beyond this, we hope that we have left some interesting open
 questions for others to examine. To this end, supplementary ma terials related to this article include:

<ul><li>Appendix A: Some answers to the question 3. (langren-app.pdf)</li><li>La Verdadera: A transcription and Engl</li></ul>	-
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These and other documents related to van "secret of Longitude," have also been dependent. <i>yorku.ca/SCS/Gallery/langren/</i> .	
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