Common Angle Plots as Perception-True Visualizations of Categorical Associations

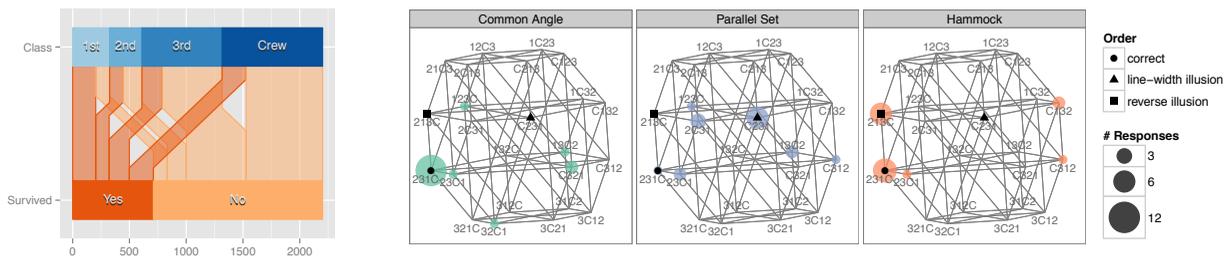Heike Hofmann and Marie Vendettuoli

Fig. 1. Who survived on the Titanic? Common Angle plot of survival by class (left). Graphs showing survey results of answers (right).

Abstract—Visualizations are great tools of communications–they summarize findings and quickly convey main messages to our audience. As designers of charts we have to make sure that information is shown with a minimum of distortion. We have to also consider illusions and other perceptual limitations of our audience. In this paper we discuss the effect and strength of the line width illusion, a Müller-Lyer type illusion, on designs related to displaying associations between categorical variables. Parallel sets and hammock plots are both affected by line width illusions. We introduce the common-angle plot as an alternative method for displaying categorical data in a manner that minimizes the effect from perceptual illusions. Results from user studies both highlight the need for addressing line-width illusions in displays and provide evidence that common angle charts successfully resolve this issue.

Index Terms—Linewidth illusion, data visualization, high-dimensional displays, parallel sets, hammock plots, Müller-Lyer illusion

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1 INTRODUCTION

A well-designed graph is a powerful tool that transcends barriers of language to communicate complex concepts from author to audience. Problems arise when readers are unable to easily extract a chart's main message or are led to wrong conclusions due to distortions. Distortions endanger the trust between readers and creators of charts. This trust is based on the premise that graphics have to be true to the data [34, 35, 28]. There is a lot of discussion on keeping true to the data in the framework of (ab)using three dimensional effects in graphics. Tufte [34] goes as far as defining the *lie-factor* – the ratio of the size of an effect in the data compared to the size of an effect shown in the chart. Any large deviation of this factor from 1.0 indicates a misuse of graphical techniques. Computational tools help us ensure technical trueness – but this brings up the additional question of how we deal with situations that involve innate inability or trigger learned misperceptions. One example of distortions of this kind is the Müller-Lyer family of illusions, which include contextual illusions, such as differently perceived lengths of line segments depending on the orientation of arrow heads or the sine illusion as defined by Day and Stecher [10].

Regardless of the cause of distortion, it is the responsibility of the author of a chart to create visualizations that allow readers to extract an accurate interpretation of the underlying data. In order to gauge the extent of distortion due to perceptual limitations, we can employ user studies to provide empirical evidence supporting underlying cognitive models or to reveal previously unknown or not anticipated illusions.

Parallel sets (parsets) as introduced by Kosara et al [23] are a graphical method for visualizing multivariate categorical data. Since their initial publication, parallel sets have spread to mass media outlets [22, 6, 3], have been implemented in various languages [22, 4, 8] and spurred further academic work [23, has 70 citations according to Google scholar]. While retaining the ability of parallel coordinates [19, 36] to visualize a large number of dimensions simultaneously, parallel sets incorporate a frequency scale that is a well-known feature of other categorical displays such as barcharts or mosaic plots [15, 12, 17, 33].

Parsets also allow the visualization of hierarchies [31, 32]. They share this ability with mosaicplots and their more general relatives, the treemaps, as introduced by Shneiderman [30], and treemap variants [5, 21, 1].

All of these properties make parsets a powerful visualization in our toolkit, but unfortunately, the parallel set plot is a victim of distortion due to a contextual illusion: consider the parset plot of Figure 2.

This plot shows the relationship between class status and survival on board the HMS Titanic (data assembled by Dawson [9] from insurance and boarding documents). The top bar in figure 2 shows the variable Class, recorded as either crew member or passenger in first, second, or third class. The bottom bar shows survival as yes and no. Lines are drawn between top and bottom bar – the (horizontal) width is proportional to the number of survivors and non-survivors they represent. A reasonable task based on this chart is to ask the reader to order class levels according to their number of survivors. However, when study participants were asked to perform this task, only 6.2% of all respondents selected the correct order, see table 3. No less than 37.5% of all participants agreed on one specific, incorrect ordering.

This phenomenon can be explained by the *line width illusion*. The

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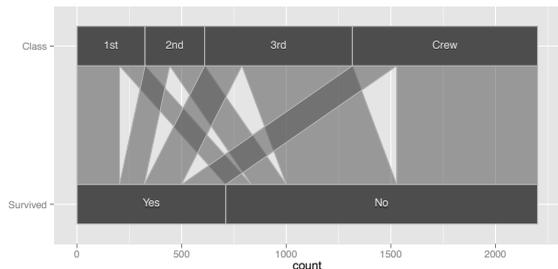
Fig. 2. Parallel sets plot showing the relationship between survival of the sinking of the HMS Titanic and class membership. Class membership and survival are clearly related, but which class had the largest number of survivors?

Table 1. Survival status and class membership of all persons on board the HMS Titanic. Most survivors were among crew members, followed by first, third, and, lastly, second class passengers.

	Crew	1st	2nd	3rd
Survivors	212	203	118	178
Non-Survivors	673	122	167	528

line-width illusion is a contextual illusion that leads to perceptual distortion in evaluating parallel sets plots. In this paper, we first describe and then quantify this illusion. We also propose and test *common angle plots* as an alternative method for visualizing multivariate categorical data that helps the audience to avoid the distortional effects of the line width illusion.

2 LINE WIDTH ILLUSIONS

The phenomenon of the line width illusion is known and widely discussed in statistical graphics literature [7, 34, 35, 28]. It is due to our tendency to assess distance between curves as the minimal (orthogonal) distance rather than the vertical distance – see sketch 5 for a visual representation of both.

On of the earliest examples of the line width illusion is shown in figure 3. This chart displays the balance of trade between England and the East Indies as demonstrated by William Playfair in his Commercial and Political Atlas, 1786 [25, 26]. One purpose of this chart is to highlight the difference between imports and exports in a particular year and the pattern of these differences over time. The difference in exports and imports is encoded as the vertical difference between the lines. When observers are asked to sketch out the difference between exports and imports (Cleveland and McGill [7]), they very often miss the steep rise in the difference between the lines in the years between
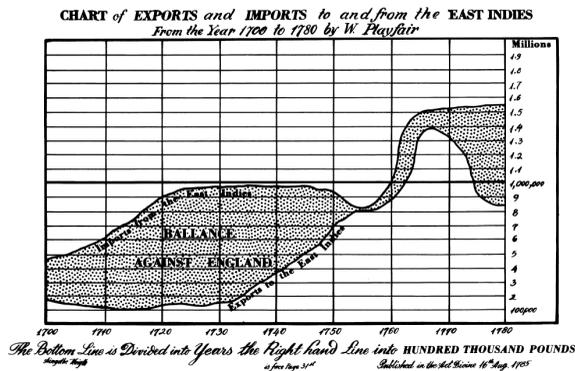

Fig. 3. Playfair's chart from the Commercial and Political Atlas (1786) showing the balance of trade between England and the East Indies. In which years was the difference between imports and exports the highest?
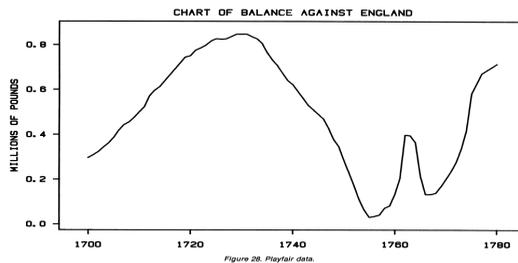

Fig. 4. Difference between exports and imports from England to and from the East Indies in the 18th century – the steep rise in the difference around 1760 comes as a surprise to many viewers of the raw data in figure 3.

about 1755 and 1765. Figure 4 shows the actual difference between imports and exports.

In the perception literature, this phenomenon is known as part of a group of geometrical optical misperceptions of a context-sensitive nature classified as Müller-Lyer illusions [10, 13]. Interestingly, there seems to be a general agreement that this illusion exists, but a quantification of it is curiously absent from literature.

The type of chart as shown in figure 3 proposed by Playfair is a quite common occurrence, particularly in election years – where these kind of charts are used to enable comparisons of support for different candidates. The recommendation from literature is to avoid charts in which the audience is asked to do visual subtractions, and show these differences directly [7, 35, 34].

2.1 Strength of the line width illusion

When visually evaluating lines of thickness greater than one, the line width illusion applies. As above, there is a strong preference of evaluating the width of lines orthogonal to their slopes as opposed to horizontally (see figure 5), which would lead us to a correct evaluation of parallel sets-style displays.

Orthogonal w_o and horizontal w_h line widths are related – the orthogonal line width depends on the angle (or, equivalently, the slope) of the line:

$$w_o = w_h \sin \theta, \tag{1}$$

where θ is the angle of the line with respect to the horizontal.
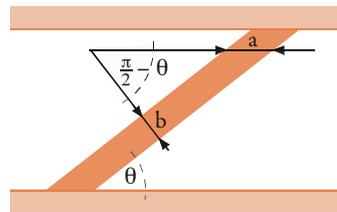

Fig. 5. Sketch of line width assessments: (a) is showing horizontal width, (b) shows width orthogonal to the slope. Survey results in section 4.2 indicate that observers associate line width more with orthogonal width w_o (b) than horizontal width w_h (a).

The perceived slope of a line depends on the aspect ratio of the corresponding plot – changing the height to width ratio of a display will change our perception of the corresponding line widths, if they are not adjusted for the slope [7]. This finding is not new, but its strength on our perception is surprising, as can be seen in the example of figure 6. Again, survival and class membership on the Titanic is shown; the same parallel sets plot is shown twice in this figure, but with very different aspect ratios: in the plot on the left the number of surviving 3rd class passengers seems to be about twice as big as the number of survivors among crew members, whereas in the plot on the right the lines have about equal (orthogonal) width. The numbers
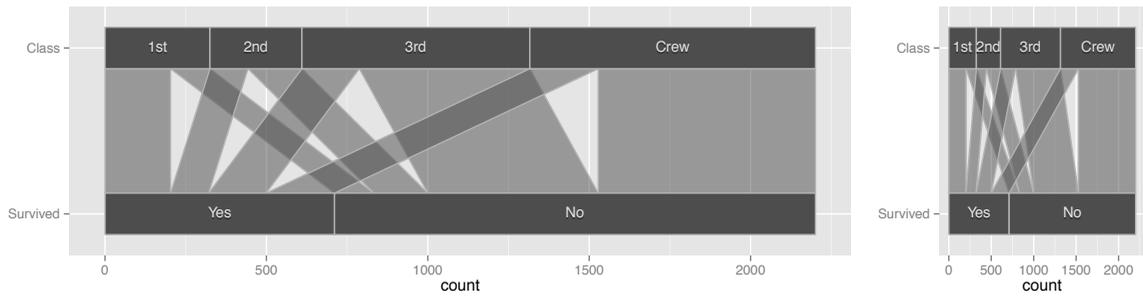
Fig. 6. Parallel sets plots of survival on the Titanic by class. Different aspect ratios seemingly change the thickness of line segments, compare e.g. number of survivors in 3rd class and in the crew.

underlying the figures are identical. Any perceived change is purely due to the different aspect ratios.

For parallel sets-style displays, the audience has the *area of the line segment* as an alternate visual cue when evaluating frequencies. Because height (or width for a rotated display) of line segments is constant across the display, the width of a particular segment is proportional to its area. We could therefore employ area comparisons as a proxy or to augment line width evaluations. However, existing literature suggests that this method of comparison is particularly prone to errors in two scenarios commonly seen in parallel sets: (1) for extreme aspect ratios of the rectangular shape (Heer et al [16]) and (2) when comparing rectangles rotated relative to each other (Kong et al [21]). This incorrect perception and comparison of areas distorts the message readers discern from the graph.

2.2 Hammock Plots

Hammock plots, introduced by Schonlau [29], provide an alternative to parallel sets that is adjusted for the line width illusion. This is done by adjusting the *horizontal* line width by a factor of $\sin \theta$, as discussed in equation (1). This adjustment makes the perceived *orthogonal* line width proportional to the number of observations it represents. Figure 7 shows an example of a hammock plot of the Titanic data. From top to bottom Class, Gender, Survival, and, again, Class are shown, capturing all two-way relationships of these variables.

The structure of a hammock plot is the same as that of a parallel sets plot; bars are divided according to class membership numbers, and lines connect categories between neighboring variables. Now *orthogonal* line widths represent the number of individuals in each combination. Additionally, the lines start from the middle of the bin and connect to the middle of the other variable's bins. This convention is in part due to the fact that the sum of horizontal widths (w_h) after adjustment is greater than the width of marginal bars.

The graph of figure 7 shows that barely any women were in the crew, while male crew members make up the second largest contingent overall (only male non-survivors are more frequent). One problem that arises when dealing with hammock plots, is that the adjustment of lines depends on the knowledge of the angle θ under which each line is drawn. This depends on the aspect ratio of the plot. Any changes to the aspect ratio, such as a non-proportional resizing of a figure results in a change of the angle and results in a re-introduction of the linewidth illusion.

2.3 Reverse linewidth illusion

A problem that arises in evaluating hammock plots is that if an observer focuses on horizontal line width the plots suffer from a *reverse* of the *line width illusion*: judging the number of survivors by class in figures 7 and 8 based on horizontal line width results in an ordering of (largest to smallest) Crew, 3rd, 1st, and 2nd – which is not the correct ordering: there were actually more survivors in the 1st class than in the 3rd class. Unfortunately, the problem of the reverse linewidth illusion is exacerbated in hammock plots due to their construction: the centering of lines around the middle of each level introduces a contextual coordinate system that encourages comparisons of horizontal
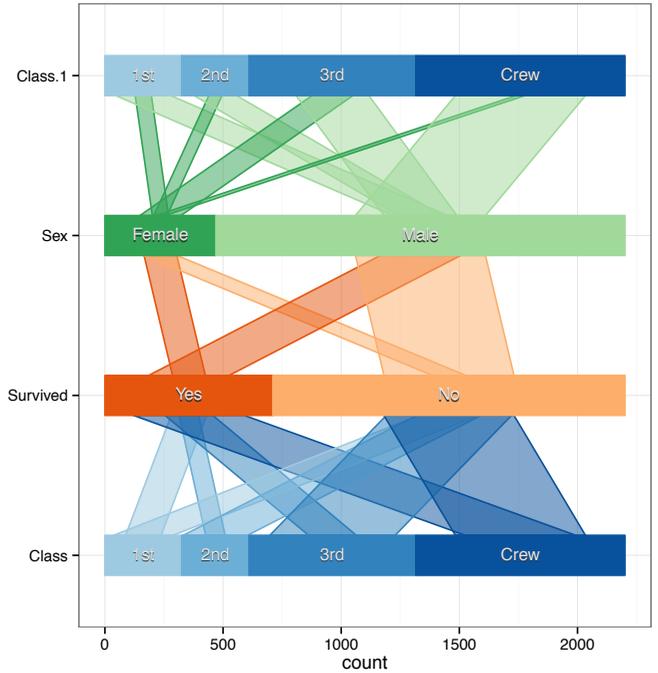

Fig. 7. Hammock plot of the relationship between Class and Survival on the Titanic.

line widths, which are due to the adjustment no longer proportional to the number of records they represent. The amount of the perceived distortion can be quantified by rearranging equation 1 as:

$$w_h = w_o \csc \theta, \qquad (2)$$

where w_o is proportional to observations and θ is again the angle of the line with respect to the horizontal.
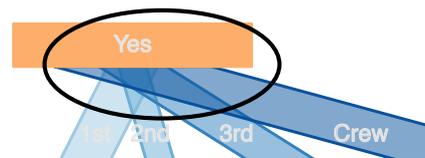

Fig. 8. Lines in hammock plot of Titanic data for survival variable, level yes. Comparing horizontal widths suggests that a greater number of survivors were from third class instead of first, which is inconsistent with underlying data.

3 COMMON ANGLE PLOTS

The idea behind the common angle plot is to construct a plot that is safe from both the line width illusion and its reverse, while keeping the basic structure of parallel sets plots and hammock plots. The line width illusion is triggered by differences in angles under which lines are drawn, so the basic idea is to draw all lines in a plot under the same angle. As this is not generally possible, a solution is to give up on the idea of straight lines between variable bars and think of ribbons instead, that are constructed from connecting line segments. We can then ensure that all of these ribbons have at least one line segment that is drawn under the same angle. Assuming horizontal bars for the variables, connecting bands between categories are drawn as a combination of a vertical segment, a segment under a pre-specified angle θ, followed by another vertical segment as sketched out in figure 9
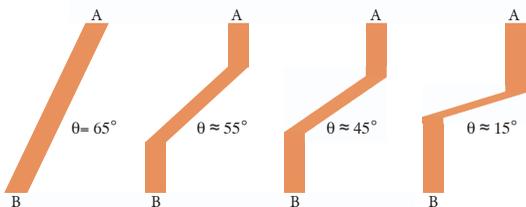

Fig. 9. Sketch of ribbons under three different angles (from left to right). All ribbons form a connection between points A and B.
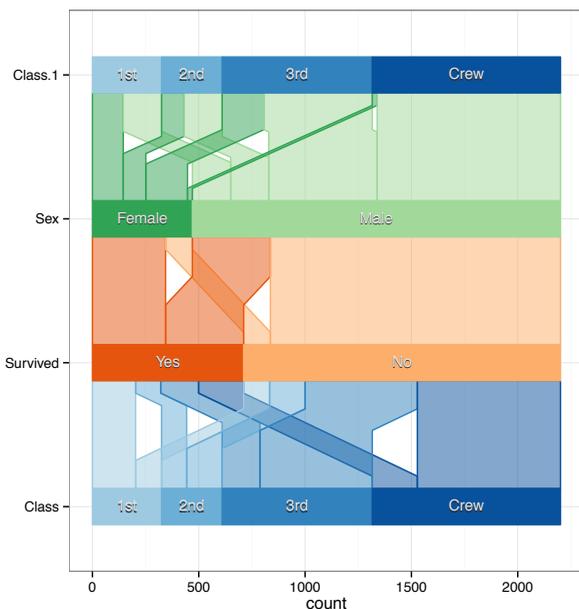

Fig. 10. Common angle plot of the Titanic data.

The pre-specified angle θ (between the line and the horizontal band) is given as –at most– the angle of the longest connecting line between two categories of neighboring variables. This makes the width of ribbons comparable without being affected by the distortion, as all ribbons are sharing at least one segment under the same angle.

Figure 10 shows a common angle plot of the same data as the hammock plot. Now, tasks such as ordering levels according to the numbers they represent are not affected by either line width illusion and 13 out of 18 participants in the survey gave a correct assessment of the number of survivors by class (more details to follow in the next section). We also see that a few more men survived than women, but proportionally the situation is very different – a much higher percentage of women survived than men. While more first class passengers survived than not, the survival chances of second class passengers tilt

towards doom. More members of the third class and the crew perished than survived.

Common angle plots, as well as all of the related methods of hammock plots and parallel sets are implemented in the package ggparallel based on the ggplot2 (Wickham [37]) plotting framework in the software R 2.15.1 [27]. The ggparallel package is freely available from CRAN (http://www.r-project.org/). The colors for the plots have been chosen using color schemes suggested by the ColorBrewer project (Harrower et al [14]), as implemented in the R package RColorBrewer (Neuwirth [24]).

4 USABILITY TESTING

4.1 Study Design

To determine the effectiveness of the common angle plot, we conducted a user study in the form of a survey asking participants to provide responses regarding the structure in two data sets with predominantly categorical variables. The Titanic data includes class, sex, age, and survival status for each person on board of the Titanic [9]. The gene data was retrieved from the UCSC Genome Browser (Kent et al [20]) and includes chromosome location for genes involved in one of three metabolism pathways: steroid biosynthesis, caffeine metabolism and drug metabolism. For each data set, participants were asked to provide responses for three tasks that analysts routinely perform as part of exploratory data analysis:

Task I: simple comparison task, chosen to be unaffected by any illusion. Performance on this task should be comparable across designs.

Task II: simple ordering, involving three pairwise comparisons, some of which are affected by the line width illusion or its reverse.

Task III: more complex ordering task with at least six pairwise comparisons, some of which are affected by either illusion.

The study was conducted in form of a crossover design (see Table 2): each participant was presented with two out of the three display types, where the first display showed the Titanic data, and the second display showed the gene data set. All participants were asked to answer the same set of questions (see Appendix A) covering tasks I through III for each data set. This design allows for comparisons of display types and tasks while it is possible to simultaneously adjust for individuals' different skill sets and learning effects.

At the start of the survey, participants were given a link to a brief tutorial regarding the different plot types. Not all of the participants followed this link. The decision to not require participants to go through a thorough training beforehand was conscious. The main goal of our study was to assess performance of the plots based on intuitive evaluation. We therefore refrained from any coaching on how to evaluate plots in the training material and restricted ourselves to an explanation of the construction.

The choice to show only two of the three possible types of displays to a participant was made to encourage participation by reducing the amount of time needed for its completion. On average, participants needed 18 minutes to complete the survey. We did not find any significant differences between the amount of time needed between the first and the second block of questions ($F_{1,95} = 0.0556$ for a p-value of 0.8142), nor were there significant differences in the length of time taken between the three designs ($F_{2,94} = 0.1909$ for a p-value of 0.8265).

No personally identifiable information was collected, nor did we offer any compensation for participation in the survey.

4.2 Results

We are investigating four aspects of the experiment in this section: **(i)** participants' performance on each task according to the percentage of correct responses, **(ii)** extent of variability due to subject-specific abilities, **(iii)** space of answers for the more complex ordering task III, and **(iv)** participants' preferences after completing tasks on two designs.

Table 2. Overview of study design and participation numbers. The number in parenthesis indicates the number of participants completing the first block, but not the second.

Titanic Data	PS	CA	H	PS	CA	H
Genes Data	CA	PS	PS	H	H	CA
#responses	8 (9)	6 (7)	8 (9)	6 (7)	10 (11)	8 (8)

Correctness of Answers

Answers for each survey question were recoded in binary form according to correctness (with 1 for correct answers, and 0 otherwise). This forms the basis for the evaluation of performance of the different designs.

Table 3 shows percentages of correct answers for each question under each design. Bold numbers indicate significantly different (worse) performance of a design compared to the common angle plot based on a generalized linear model with random effects to adjust for individuals' abilities. The model explains 77.4% of the total variability, corresponding to a highly significant statistic of 452.7 (p-value $\ll$ 0.0001).

The observed results match our expectations: task I does not show any significant differences between the designs and has overall the highest percentage of correctness reflecting its low difficulty level. Generally, the percentage of correct responses for a task decreases with an increase of its complexity.

Parallel sets were affected the most by the line width illusion and show significantly worse performance for tasks II and III in both the Titanic and the genes data.

Hammock plots led to significantly worse performance than common angle plots in the two questions that were affected by the reverse line width illusion, while they show equal performance as common angle plots for the other questions. For task II in the genes data, hammock plots have the overall best performance across designs– but this does not constitute a significant improvement over the performance of the common angle plot. Only for a sample size of over 150 participants would this difference be significant (for a power of 50% to detect this difference). The pathway-chromosome relationship is shown in a common angle plot in figure 12. The top bar identifies chromosome location, the bottom pathways. Ribbon widths visualize the number of genes on a chromosome that are active in a particular pathway. Task III for the genes data asked for an ordering of the chromosomes involved in the pathway of steroid steroid.

Figure 11 gives an overview of the performance of each design on all tasks.

Table 3. Percentages (standard deviation) of correct responses for each task and design. Bold numbers indicate significant difference from common angle plot performance.

Task	Data	Design		
		CA	H	PS
I	Titanic	85.2 (0.66)	76.5 (0.84)	68.8 (0.98)
	Genes	93.8 (0.51)	83.3 (0.78)	83.3 (0.90)
II	Titanic	72.2 (2.56)	**17.6** (2.31)	**25.0** (2.80)
	Genes	75.0 (2.80)	87.5 (2.13)	**57.1** (3.67)
III	Titanic	66.7 (2.69)	**41.2** (2.98)	**6.2** (1.56)
	Genes	68.8 (2.99)	68.8 (2.99)	**7.1** (1.91)

Individuals' skill levels

Figure 13 shows an overview of the predicted skill for each participant under the model. Skills are quite varied between -1.52 and 1.34, but a Kolmogorov-Smirnov test does not show significant deviation from a normal assumption (p-value 0.089). On the scale of the dependent variable the range in individuals' skills translates to a $17.5 = e^{1.34 - (-1.52)}$ fold increase in the probability of answering a question on the survey correctly between participants with the best skill set and the worst.
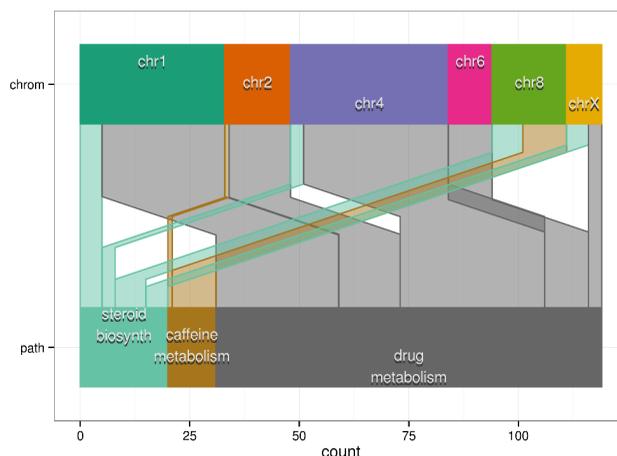

Fig. 12. Common angle plot of the genes data. The top bar shows chromosome, the bottom the pathway. Width of lines corresponds to the number of genes pathways and chromosomes have in common. Both drug metabolism (hsa00982) and caffeine metabolism (hsa00232) make use of genes on chromosome 2. Task III of the survey asked for the order of the chromosomes involved in the steroid biosynthesis pathway from fewest number of genes to highest.
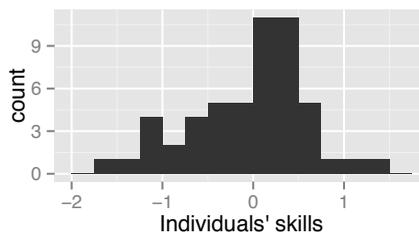

Fig. 13. Histogram of the predictions of subject-specific skills.

Evidence for line width illusions

Task III for the Titanic data required participants to order class levels according to the number of survivors, fewest to highest.

There are 4! = 24 distinct orderings of the levels, corresponding to all permutations of length four. Some orderings are closer to one another than other orderings. The Cayley distance allows us to quantify this distance; the Cayley distance between two orderings is defined as the smallest number of switches necessary to get from one ordering to the other. Visually, this corresponds to a graph; each node represents one ordering, and two nodes are connected by an edge, if only a single switch is necessary to move from one ordering to the other, i.e. if the Cayley distance between these nodes is one. This results in a regular graph of degree six, where each node is connected to six other nodes. Between any two nodes, the Cayley distance on the graph is equivalent to the length of the shortest connecting path between the two nodes. Figure 14 shows an overview of the permutation space together with an overview of the survey results.

The colored dots on top of the graph correspond to the responses from the survey. The size of these dots is proportional to the number of observers choosing this particular ordering. It becomes obvious from the three graphs in figure 14 that the answers to different designs occupy quite different regions, while answers based on the same design are quite close and usually separated by only one edge.

The correct ordering, as well as the orderings assuming the line width illusion and its reverse are marked by symbols. Answers for the common angle plot are centered around the correct answer, while responses to parallel sets cluster around the response corresponding to the line width illusion. Answers based on the hammock design are split evenly between the correct answer and the answer corresponding
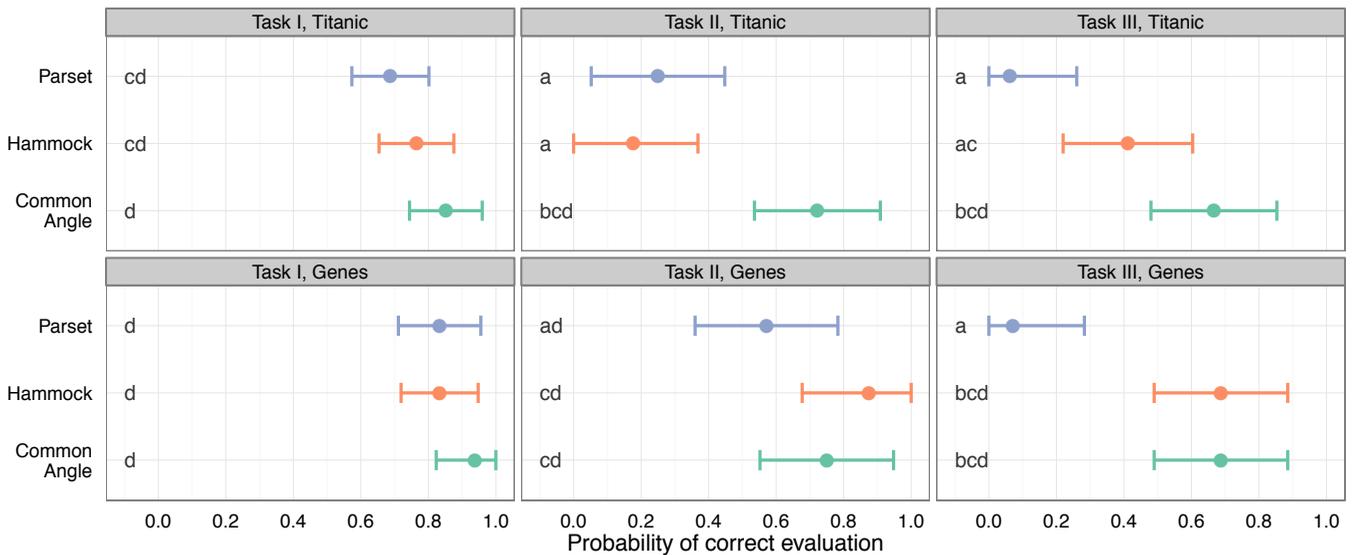
Fig. 11. Overview of performance across tasks and designs. Points show average performance of subjects on each of the tasks, lines represent 95% confidence intervals adjusted for multiple comparisons. The letter at the front of each panel allows for an evaluation of significance of pairwise comparisons: if two averages do not share a letter, they are significantly different at a level of 0.05.

to the inverse line width illusion. Table 4 gives an overview of all responses to task III for the Titanic data.

Table 4. Responses to task III in the Titanic data: order levels of Class by the number of survivors (smallest to largest).

Order	CA	H	PS	
Crew, 1st, 3rd, 2nd		2		
2nd, 1st, 3rd, Crew		6		reverse line width illusion
Crew, 3rd, 1st, 2nd		1	1	
2nd, 3rd, 1st, Crew	12	7	1	correct
2nd, 3rd, Crew, 1st	1	1	2	
Crew, 3rd, 2nd, 1st	2			
3rd, 2nd, Crew, 1st	1			
1st, 2nd, 3rd, Crew	1		1	
1st, 3rd, Crew, 2nd	1		2	
Crew, 2nd, 3rd, 1st			6	line width illusion
2nd, Crew, 3rd, 1st			3	
Total	18	17	16	

Common angle plots show the best performance in terms of correctness (66.7% on 18 responses), compared to a correctness of 6.2% for the parallel sets plot on 16 responses, constituting a significantly better performance of the common angle plot at a level of < 0.0001, based on a Mantel-Haenszel test (the difference in performance to the hammock plot is not significant with p-value of 0.1359, but the hammock plot performs also significantly better than parallel sets plot with a p-value of 0.0016). While the intuitive assessment of lines by their width orthogonal to their direction is well known, it is surprising to see its strength: in this particular setting, it is strong enough to 'shrink' the horizontally widest line for six out of 16 participants by at least 44%, from 212 to below 118, and a further three participants perceived a shrinkage to below 178, a distortion factor of at least 16%.

Opinion on common angle plots

Answers to the question of 'which chart did you like better?' are shown in table 5. There is a clear endorsement in favor of common angle plots versus the other two types of displays. The most common reason cited for the choice was a facilitated comparison of width, area or "size", The only consistent complaint against common angle was a preference for straight lines. This purely aesthetic preference is deeply rooted and in our opinion the biggest challenge for common

angle plots.

Table 5. Preferences for first or second chart across all six combinations of questions and chart types.

			Which chart did you like better?	
			Chart 1	Chart 2
PS	vs	**CA**	2	**6**
CA	vs	PS	**4**	2
H	vs	**CA**	3	**5**
CA	vs	H	**8**	2
H	vs	PS	3	5
PS	vs	H	1	5

4.3 Methods

The survey was created using the Qualtrics Labs, Inc software (www.qualtrics.com). For survey contents, see Appendix A. The study design is presented in section 4.1. All models are fit in the `lme4` package (Bates et al [2]) within the software framework of R 2.15.1. Comparisons are adjusted for multiple testing using the `multcomp` package (Hothorn et al [18])and evaluated for pairwise significances using the `effects` package (Fox [11]).

5 DISCUSSION

There is strong support from the user study that common angle plots help the reader to overcome issues arising from the line-width illusion and its reverse. This might come as a surprise, in particular, as common angle plots break one of the nice mathematical properties that parallel sets have: the area of a connecting line in parallel sets has a constant overall area independent of the angle under which it is drawn. Both hammock plots and common angle plots break this property. It does not seem, however, that the audience picks up on area as the main property of the displays.

There are several other issues that common angle plots do not address in the visualization of categorical data, that should be noted at this point:

- Large number of levels in a variable introduces a lot of line crossings, which affects the overall effectiveness of the display. The number of line crossings is the same in parallel sets plots, but hammock plots reduce the number of crossings by centering all lines.
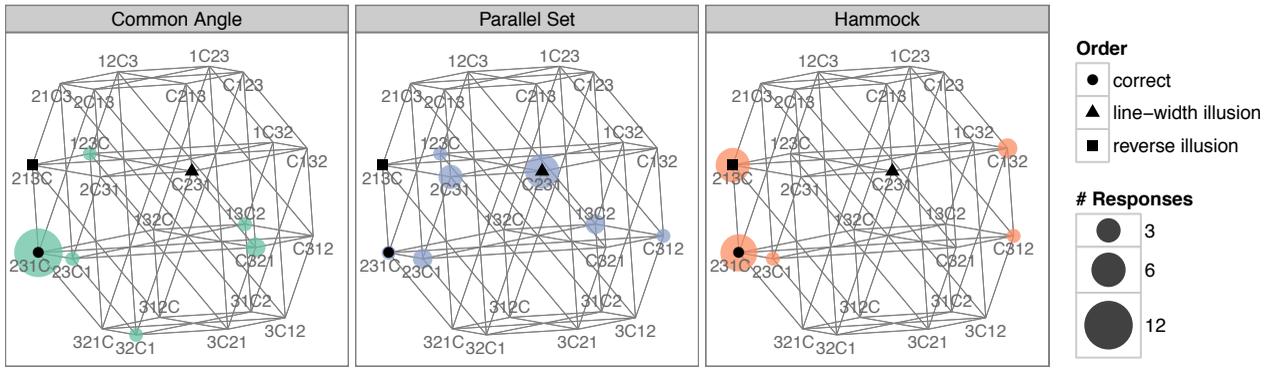
Fig. 14. Answers to task III in the Titanic data – each node corresponds to a single ordering of the levels in variable 'Class'. Lines are drawn between orderings that are only one swap of levels apart. The colored dots show responses from the survey, their sizes depend on the number of responses for each ordering.

• The use of color to separate levels is also problematic for large number of categories in a variable, as it leads to palettes with very similar colors.

Apart from these problems that will have to be solved in a different framework of plots, there are several opportunities for extending common angle plots. One opportunity for extension lies in the algorithm to determine the thickness of the connecting line. In the tested version of common angle plots, the line width was not explicitly defined - the line width is a byproduct of specified θ for a band that connects marginal bars. Using an additional hammock adjustment as given in equation 1 in the slanted section of the line segment, we can keep the bandwidth constant, resulting in a common angle-hammock plot hybrid that bears further investigation. A drawback of this approach is that the sum of the bandwidths w_h will no longer match the widths of the marginal bars, which might create an additional processing burden for the audience in mapping the relationship between band width and the width of marginal bars. Both hammock plots and this modification of common angle plots face the issue of band *area* as context to support reader interpretation of w_o. Since the band area is now related to the incident angle θ, changes in the display aspect ratio may have a distortion effect. In the study described in this paper, this effect was not evaluated and aspect ratios were kept constant.

In the original paper (Kosara et al [23]), parallel sets were introduced to reflect a hierarchy of variables. Prior examples in this paper show sets of two-dimensional plots to focus on the association between pairs of variables. With color coding, it is possible to show hierarchies in all of the types of displays. Figure 16 shows a common angle plot with a hierarchy: survivors of the disaster are marked in blue, non-survivors by orange. From top to bottom of the plot a hierarchy is drawn, considering first survival, then gender, followed by age and finally class membership. The coloring tracks survival status throughout the hierarchy, the layout in a common angle plot makes comparisons valid across all levels. This is of particular importance in hierarchical displays, which by definition have a larger number of smaller groups than displays without a hierarchy exacerbating problems induced by the line width illusion.

Another opportunity for extending common angle plots is to add interactivity. It is important to note that any additions of functionality via interactivity should not come at the expense of developing distortion-free displays. In particular, a display with visual cues in conflict with (textual) interactive feedback introduces a higher cognitive load by asking the audience to make a decision between one of the sources of information (and disregard the other one as incorrect).
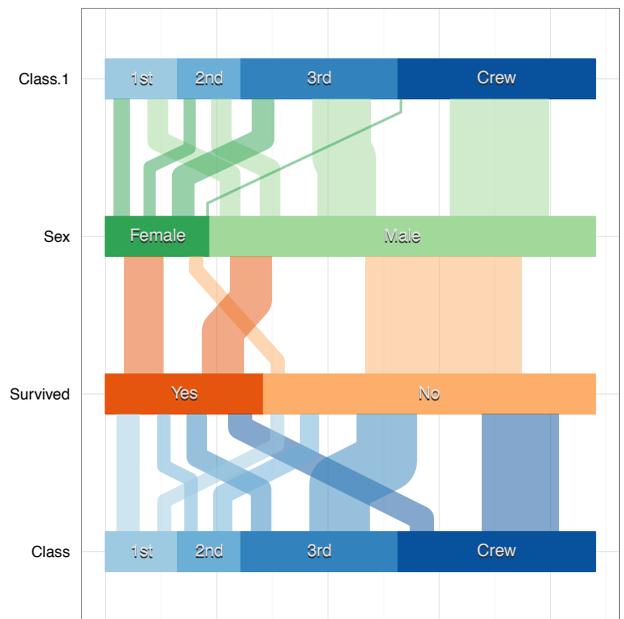

Fig. 15. Common angle plot of Titanic data using hammock correction.

6 CONCLUSION

We have proposed a new chart type for visualizing multivariate categorical data, common angle plots, and tested its usability compared to existing charts that perform a similar function. Results from user testing indicate that common angle plots effectively communicate underlying data without being affected by perceptual distortions due to the line width illusion or its reverse.

Two other chart types which address visualization of multivariate categorical data: parallel sets and hammock plots, are subject to the line width illusions. Audiences perceive parallel sets with distortion due to a natural tendency to evaluate line width in the orthogonal direction while data is mapped to the horizontal width. For hammock plots a correction is made to map data to the orthogonal width, however the centering of the lines creates a strong contextual cue that encourages an evaluation of line widths using the horizontal measure, leading to a reverse line width illusion. Common angle plots avoid the perceptual distortion associated with either version of the illusion regardless of
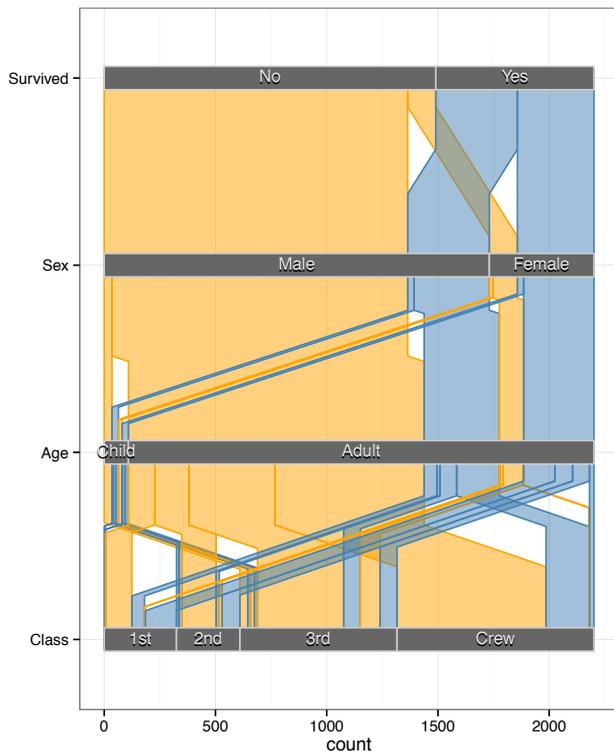
Fig. 16.   Common angle plot of the Titanic data using a hierarchical structure in the variable (cf. to parallel sets chart in [8]).

the underlying data set.

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A   SURVEY QUESTIONS

The tutorial shown to participants at the start of the survey can be found at `http://mariev.net/tutorial.html`.
The questions pertaining to the Titanic data were:

Task 1:   *Agree, Disagree or Don't Know/Can't Determine with the following statements:*

- There were an approximately equal number of Male and Female Survivors
- The group with largest number of travelers was Female Survivors
- There were more Male Non-Survivors than number of males in First and Second Class Combined

Task 2:   *Order the following groups by number, fewest to most*

- 1st Class female passengers
- Male Survivors
- Crew Survivors

Task 3:   *Order the categories of Class by number Survived, fewest to most.*

- 1st
- 2nd
- 3rd

- Crew

The questions pertaining to the gene data were:

Task I: *Agree, Disagree or Don't Know/Can't Determine with the following statements:*

- There are about the same number of genes in the group "steroid biosynthesis:chromosome 1" as in the group "caffeine metabolism: chromosome 8"
- The group with the greatest number of genes is "drug metabolism:chromosome 4"
- there are more genes involved in the group "drug metabolism: chromosome 1" than all genes involved in the caffeine metabolism pathway

Task 2:   *Order the following chromosomes by number of genes involved, fewest to most.*

- steroid biosynthesis :: chromosome X
- steroid biosynthesis :: chromosome 4
- drug metabolism :: chromosome X

Task 3:   *Order the following chromosomes by number of genes involved in steroid biosynthesis pathway, fewest to most.*

- chromosome 1
- chromosome 4
- chromosome 8
- chromosome X

B   PARTICIPANTS' DEMOGRAPHICS

All students, staff and faculty from Iowa State University programs in Statistics, Bioinformatics and Computational Biology and Human Computer Interaction were invited to participate by email. 93 individuals accessed the survey; 86 participants gave consent, 15 of those dropped out right after, 20 went to the training site and did not return. Out of the remaining 51 participants, 46 individuals submitted responses for all questions and five gave responses to the first block of questions.

Participants used their own personal computing devices to access the survey, a majority of participants used Intel Mac OS X (versions ranging from 10.6.8 to 10.8.2), while Windows was the next most common operating system. The preferred choice of browser was Firefox, followed by Chrome. For two participants, the Qualtrics survey software was unable to capture operating system or browser information.

REFERENCES

[1] M. Balzer and O. Deussen. Voronoi treemaps. In *Proc. IEEE InfoVis*, 2005.

[2] D. Bates, M. Maechler, and B. Bolker. *lme4: Linear mixed-effects models using S4 classes*, 2013.   R package version 0.999999-2.

[3] M. Blastland.   Go figure: How to understand risk in 13 clicks. `http://news.bbc.co.uk/2/hi/uk_news/magazine/7937382.stm`, March 11 2009.

[4] M. Bostock, V. Ogievetsky, and J. Heer. D3 data-driven documents. *IEEE Transactions on Visualization and Computer Graphics*, 17(12):2301–2309, Dec. 2011.

[5] M. Bruls, K. Huizing, and J. J. van Wijk. Squarified treemaps. In *IEEE Symposium on Information Visualization*, 1999.

[6] S. Carter and M. Bostock. Over the decades, how states have shifted? http://www.nytimes.com/interactive/2012/10/15/us/politics/swing-history.html, Oct 15 2012.

[7] W. S. Cleveland and R. McGill. Graphical perception: Theory, experimentation, and application to the development of graphical methods. *Journal of the American Statistical Association*, 79(387):531–554, 1984.

[8] J. Davies. Parallel sets. http://www.jasondavies.com/parallel-sets/, 2012.

[9] R. J. Dawson. The 'unusual episode' data revisited. *Journal of Statistics Education*, 3(3), 1995.

[10] R. H. Day and E. J. Stecher. Sine of an illusion. *Perception*, 20:49–55, 1991.

[11] J. Fox. Effect Displays in R for Generalised Linear Models. *Journal of Statistical Software*, 8(15):1–27, 2003.

[12] M. Friendly. *Visualizing Categorical Data*. SAS Press, Cary, NC, 2000.

[13] E. Goldstein. *Sensation & Perception*. Thomason Wadsworth, Belmont, CA, 2010.

[14] M. A. Harrower and C. A. Brewer. ColorBrewer.org: An Online Tool for Selecting Color Schemes for Maps. *The Cartographic Journal*, 40(1):27–37, 2003.

[15] J. A. Hartigan and B. Kleiner. Mosaics for contingency tables. In *Computer Science and Statistics: Proceedings of the 13th Symposium on the Interface*, pages 268–273, Fairfax Station, VA, 1981. Interface Foundation of North America, Inc.

[16] J. Heer and M. Bostock. Crowdsourcing graphical perception: Using mechanical turk to assess visualization design. In *ACM Human Factors in Computing Systems (CHI)*, pages 203–212, 2010.

[17] H. Hofmann. Exploring categorical data: Interactive mosaic plots. *Metrika*, 51(1):11–26, 2000.

[18] T. Hothorn, F. Bretz, and P. Westfall. Simultaneous inference in general parametric models. *Biometrical Journal*, 50(3):346–363, 2008.

[19] A. Inselberg. The plane with parallel coordinates. *The Visual Computer*, 1(2):69–91, 1985.

[20] W. Kent, C. Sugnet, T. Furey, K. Roskin, T. Pringle, A. Zahler, and D. Haussler. The human genome browser at UCSC. *Genome Research*, 12(6):996–1006, June 2002.

[21] N. Kong, J. Heer, and M. Agrawala. Perceptual Guidelines for Creating Rectangular Treemaps. *Visualization and Computer Graphics, IEEE Transactions on*, 16(6):990–998, 2010.

[22] R. Kosara. Eagereyes. http://eagereyes.org/parallel-sets, 2006–.

[23] R. Kosara, F. Bendix, and H. Hauser. Parallel sets: Interactive exploration and visual analysis of categorical data. *IEEE Transactions on Visualization and Computer Graphics*, 12(4):558–568, July 2006.

[24] E. Neuwirth. *RColorBrewer: ColorBrewer palettes*, 2011. R package version 1.0-5.

[25] W. Playfair. *Commercial and Political Atlas*. self, London, 1786.

[26] W. Playfair, H. Wainer, and I. Spence. *Playfair's Commercial and Political Atlas and Statistical Breviary*. Cambridge University Press, 2005.

[27] R Core Team. *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria, 2013.

[28] N. Robbins. *Creating More Effective Graphs*. Wiley, 2005.

[29] M. Schonlau. Visualizing categorical data arising in the health sciences using hammock plots. In *Proceedings of the Joint Statistical Meetings, Section on Statistical Graphics*. American Statistical Association, 2003.

[30] B. Shneiderman. Tree visualization with tree-maps: 2-d space-filling approach. *ACM Trans. Graph.*, 11(1):92–99, 1992.

[31] A. Slingsby, J. Dykes, and J. Wood. Configuring hierarchical layouts to address research questions. *IEEE Transactions on Visualization and Computer Graphics*, 15(6), 2009.

[32] J. Stasko and E. Zhang. Focus+context display and navigation techniques for enhancing radial, space-filling hierarchy visualizations. In *Proc. IEEE InfoVis*, 2000.

[33] M. Theus, H. Hofmann, B. Siegl, and A. Unwin. *New Techniques and Technologies for Statistics II*, chapter MANET: Extensions to interactive statistical graphics for missing values. IOS Press Amsterdam, 1997.

[34] E. Tufte. *The Visual Display of Quantitative Information*. Graphics Press, USA, 2 edition, 1991.

[35] H. Wainer. *Visual Revelations*. Psychology Press, 2000.

[36] E. J. Wegman. Hyperdimensional data analysis using parallel coordinates. *Journal of the American Statistical Association*, 85(411):pp. 664–675, 1990.

[37] H. Wickham. *ggplot2: elegant graphics for data analysis*. Springer New York, 2009.