The Language of Graphs: from Bertin to GoG to ggplot2

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Topics

• Idea: Graphs as visual language
  ▪ Early attempts at standardization of graphs
• Jacques Bertin: *Semiology of Graphics*
  ▪ Mapping of visual properties to data relations
• Graphics programming languages:
  ▪ Goal: power & elegance
• Lee Wilkinson: *Grammar of Graphics*
• Hadley Wickham: ggplot2

Metaphor: Graphs as visual language

• Playfair (1802)
  ▪ “Regarding numbers and proportions, the best way to catch the imagination is to speak to the eyes”
• Minard (1861)
  ▪ “The aim of my carte figurative is ... to convey promptly to the eye the relation not given quickly by numbers requiring mental calculation.”
• Émile Cheysson (1890)
  ▪ “When a law is contained in figures, it is buried like metal in an ore; it is necessary to extract it. This is the work of graphical representation. It points out the coincidences, the relationships between phenomena, their anomalies, and we have seen what a powerful means of control it puts in the hands of the statistician to verify new data, discover and correct errors with which they have been stained.”

1870-1910: Statistical albums

• From ~ 1870—1910, statistical albums of official statistics on topics of population, trade, moral & political issues became widespread throughout Europe and the U.S.
  ▪ France: *Album de Statistique Graphique*: 1879-1899 (trade, commerce & other topics)
  ▪ USA: Census atlases: 1870/80/90--
    ▪ Required by Congress to give proportional representation in the House of Representatives
    ▪ Designed to give a “portrait of the nation”
  ▪ Switzerland: Atlas graphique de la Suisse:1897, 1914
  ▪ Others: Germany, Latvia, Romania, Bulgaria, etc.
Need for standardization

- Beautiful graphics: Yes, but all separate designs
  - Can anything be compared across countries?
- Émile Cheysson (1878)
  - “The time will come when Science has to lay down general principles and decide on well-defined standards. We can no longer tolerate this sort of anarchy”
- International statistical meetings
  - 1852 (Brussels), 1857 (Vienna), 1869 (The Hague), 1872 (St. Petersburg), 1876 (Budapest) ...
  - Participants: Cheysson, Levasseur (France), Ernest Engel, Gustav von Mayr, Hans Schwabe (Germany), Francis Walker (U.S.), ...

No consensus

- St. Petersburg (1872) resolutions:
  - “The Congress accepts that it is not worth going into details about the choice of methods or facts for graphical representation”.
  - “no strict rule can be imposed on authors, because the only real problem is that of applying the graphical method to data that is comparable”.
- Most of the debate had to do with thematic maps
  - number of class intervals for a quantitative variable
  - number and variety of shading colors
- Yet, the idea of a visual language had been accepted, along with the need for some theory of graphs

Bertin: Semiology of graphics (1967)

- Defines a system of “grammatical elements” of graphs and relations among visual attributes that give meaning (semantics) from perceptual features
  - Planar variables: (x,y) coordinates
  - Retinal variables: shape, size, color, ...

Bertin: Semiology of graphics

- Defines a system of mapping of retinal variables to properties of data variables for perception of relations
  - Association (≡) – marks are perceived as similar
  - Selection (∈) – marks are perceived as forming classes
  - Order (O) – marks are perceived as showing order
  - Quantity (Q) – marks are perceived as proportional
- This is the first theory of graphs relating visual attributes (encoding) to perceptual characteristics (decoding).
  - It comprises nearly all known graph and thematic map types in a general system
The retinal variables and relationship types can be implanted in various symbol types in the plane (X,Y).

Visual variables & data characteristics

Visual variables differ in the kinds of information they can convey.

<table>
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<th>Selective</th>
<th>Associative</th>
<th>Quantitative</th>
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Legend:
- GEO: (x,y)
- T, V, OR: ordered

Some recommendations

Various authors have used Bertin’s system to make recommendations for the best attributes to use with different symbol types.

Retinal variables allow for one more variable to be encoded. Bertin’s system provides a general framework for thematic mapping, allowing multiple variables to be shown simultaneously in a single map.

Legend:
- GEO: (x,y)
- T, V, OR: ordered

For Bertin, the legend is a symbolic description of the coordinate system and the variables displayed.
Decoding: Reading a graphic

How successful is a graph for transmitting information? Bertin defines three **stages** for reading a graphic:

- **External**: What is the overall context?
  - Graph title, axis labels
- **Internal**: What visual variables are used to represent the components in the graphic?
  - points, lines, ...
  - size, shape, color: hue, color: intensity, texture, ...
- **Relationships**:
  - How are these components related?
  - What questions can I ask of this graphic?
  - What can I learn?

Research topic: Have there been any studies of this ordering in graph perception?

Reading levels

Questions a graph should answer:

- Elementary: find some specific value
- Intermediate: make comparisons, see a trend
- Overall: what is the general message or overall trend?

These ideas provided the beginnings of a theory of graphs related to graph perception.

Reading levels: Example

Graph from the NY Times, Feb. 3, 2016

External: "Gun sales", time, Obama, text labels

Internal: lines, points for labeled events

Relationships: what is the message?

Reading tasks:

- Elementary: “How many guns were sold in January of 2013?”
- Intermediate: “What’s the trend in gun sales since President Obama was elected?”
- Overall: “What’s the overall trend in gun sales in America since the year 2000?”

From: https://medium.com/@karlsluis/before-tufte-there-was-bertin-63af71cea62

Bertin: The reorderable matrix

A data table: objects by characteristics

Both rows and columns are reorderable (#)

Overall relation can be discovered by permuting rows, cols

Encode each value by visual attributes
The reorderable matrix

Permute rows and columns to put like with like

Interpret row/col order & clusters

This is an early example of what I called “effect ordering” for data display

This was used by Bertin and others in a large number of applied projects

Bertin was to visual data analysis in France what Tukey was to EDA in N. America

Bertifier

Bertifier: A web app implementing Bertin’s idea of the reorderable matrix

See: http://www.aviz.fr/bertifier

Heatmaps

Heatmaps are a re-invention of Bertin’s ideas:
- Cluster analysis to reorder rows/cols
- Shading cells to show some variable

This example shows a microarray analysis of 128 leukemia patients using 12625 genes.
- The goal is to distinguish two types of leukemia
- The shading variable is a z-score for how well a given gene distinguishes the two types.

table: Attitudes and attributes by country
Values encoded by size and shape
Sorted and grouped by themes and country regions

Watch: Youtube video of Bertifier, http://youtu.be/TjxAF_a_y8Q

Image source: https://warwick.ac.uk/fac/sci/moac/people/students/peter_cock/heatmap/
Heatmaps: the devil is in the details

There are many implementations of “heatmaps” They differ importantly in the details of: clustering, shading scheme

This example shows a data set of 11 measures on 32 cars from the 1974 Motor Trends magazine

- Each variable was converted to z-scores
- The value was shaded using a bipolar color scheme
- Clusters of cars are slightly separated
- The very high and low values stand out


Making graphs: menus vs. syntax

Menu-driven graphics provide a wide range of graph types, with options

What’s wrong with that?

WYSIAYG: What you see is all you get. No way to do something different
Not reproducible: Change the data → Re-do manually from scratch
Often designed by programmers with little sense of data vis

Programming languages: Power & elegance

- CS view: All programming languages can be proved to be equivalent (to a Turing machine)
- Cognitive view: Languages differ in:
  - expressive power: ease of translating what you want to do into the results you want
  - elegance: how well does the code provide a human-readable description of what is done?
  - extensibility: ease of generalizing a method to wider scope
  - learn-ability: your learning curve (rate, asymptote)

Language | Features: Tools for thinking?
---|---
FORTRAN | Subroutines – reusable code
APL, APL2STAT | N-way arrays, nested arrays
Logo | Turtle graphics
Lisp, LispStat, ViSta | Object-oriented computing
Perl | Regular expressions, Search, match, transform, apply
SAS | Data steps, PROC steps, BY processing
R | Object-oriented methods, tidyverse: dplyr, ggplot2, ...
Programming languages: Elegance - **Logo**

Features:
- Based on Lisp, but tuned to young minds
- Turtle graphics: draw by directing a turtle, not by \((x,y)\) coordinates
  - Analytic geometry rests on a coordinate system.
  - Turtle geometry is “body syntonic”: Tell turtle what to do.
- Data types:
  - words, lists, arrays, property lists
- Lists & list processing: inherited from Lisp, but with gentler syntax.
  - Lists are infinitely expandable & nestable.
- Recursion rather than iteration is the natural method to process lists
- Extensions:
  - multiple, animated turtles (sprites);
  - object-oriented programming (message passing) -> SmallTalk

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Logo : Turtle graphics

Turtle primitives: forward, back, left, right, penup, pendown, ...

```
> spiral :size :angle  if :size > 100 [stop]  forward :size right :angle螺旋 (:size + 2) :angle end
```

Recursive procedures:
```
> spiral 0 90
> spiral 0 91
```

Logo procedures: teach the turtle a new word
```
> to square :side
repeat 4 [fd :side rt 90] end
```

```
> square 100
```

---

Logo : Hilbert curves

Logo was more than just pretty pictures
It was graphics & mathematics for young minds: A language for learning

Start with some basic shape
What happens if you replace each line with a smaller copy of the basic shape?
What happens if you continue this process?
What happens if you choose a different basic shape?

```
to Hilbert0 :turn :size
right :turn forward :size left :turn forward :size right :turn end
```

```
to Hilbert :depth :turn :size
if :depth = 0 [stop] right :turn
Hilbert (:depth-1) -:turn :size forward :size left :turn
Hilbert (:depth-1) :turn :size forward :size
Hilbert (:depth-1) :turn :size forward :size left :turn
Hilbert (:depth-1) -:turn :size forward :size right :turn end
```

Hilbert curve: A continuous, space-filling fractal, of Hausdorff dimension 2

Theorem (Hilbert, 1891): The euclidean length of the \(n\)-th depth Hilbert curve, \(H_n\) is
```
2^n - \frac{1}{2^n}
```

Proof (by enumeration): Redefine forward to calculate total turtle path length
```
to forward :length :size
make "total.length :total.length + :size forward :size end
```
The Tower of Hanoi problem has an elegant solution in Logo. Change the ‘move’ instruction to render on screen or by a robot!

Graphics programming languages: SAS

- **SAS**: procedures + annotate facility + macros
  - PROC GPLOT (x,y plots), PROC GCHART, PROC GMAP, ...
  - Annotate: data set with instructions (move, draw, text, fonts, colors)
  - Macros: Create a new, generic plot type, combining PROC steps and DATA steps.

Why I gave up SAS: This works well, and is very powerful, but lacks elegance.

Wilkinson: Grammar of Graphics

- **Natural language**:
  - Grammar/syntax: What are the minimal, complete set of rules to describe all well-formed sentences?
    - John ate the big red apple
    - John big apple red apple ate the
  - Semantics: How to distinguish meaning, nonsense, poetry in well-formed sentences?
    - Large green trucks carry garbage
    - Colorless green ideas sleep furiously
- **How to apply these ideas to graphics?**
  - Grammar: Algebra, scales, statistics, geometry, ...
  - Semantics: Space, time, uncertainty, ...
  - Needed: a complete formal theory of graphs & computational graphics language

Wilkinson: Grammar of Graphics

- A complete system, describing the components of graphs and how they combine to produce a finished graphic
  - “The grammar of graphics takes us beyond a limited set of charts (words) to an almost unlimited world of graphical forms (statements)” (Wilkinson, 2005, p. 1).
  - “… describes the meaning of what we do when we construct statistical graphics … more than a taxonomy”
  - “This system is capable of producing some hideous graphics … This system cannot produce a meaningless graphic, however.”
- This is a general theory for producing graphs.
  - the foundation of most modern software systems;
  - not connected with a theory for reading graphs à la Bertin.
Wilkinson: Grammar of Graphics

- Components:
  - **specification**: a formal language for composing graphs
  - **assembly**: coordination of attributes
    - internal: a data structure for a graphical “object”
  - **rendering**: producing a graphic on a display system
    - low level: device drivers for screen, PDF, PNG, SVG, ...

- Specification
  - **Algebra**: combine variables into a data set to be plotted
    - cross (A\(\times\)B), nest (A/B), blend (A+B), filter, subset, ...
  - **Scales**: how variables are represented
    - categorical, linear, log, power, logit, ...
  - **Statistics**: computations on the data
    - binning, summary (mean, median, sd), region (CI), smoothing

Grammar of Graphics: Specification

- **Geometry**: Creation of geometric objects from variables
  - Functions: point, line, area, interval, path, ...
  - Partitions: polygon, contour,
  - Networks: edge
  - Collision modifiers: stack, dodge, jitter
- **Coordinates**: Coordinate system for plotting
  - transformations: translation, rotation, dilation, shear, projection
  - mappings: Cartesian, polar, map projections, warping, Barycentric
  - 3D+: spherical, cylindrical, dimension reduction (MDS, SVD, PCA)

- Aesthetics: mapping of qualitative and quantitative scales to sensory attributes (extends Bertin)
  - Form: position, size, shape (polygon, glyph, image), rotation, ...
  - Surface: color (hue, saturation, brightness), texture (pattern, orientation), blur, transparency
  - Motion: direction, speed, acceleration
  - Sound: tone, volume, rhythm, voice, ...
  - Text: label, font, size, ...
- **Facets**: Construct multiplots (“small multiples”) by partitioning, blending or nesting
  - **Guides**: Allow for reading the encodings of variables mapped to aesthetics
    - scales: axes, legend (labels: size, shape, color, ...)
    - annotations (title, footnote, line, arrow, ellipse, text, ...)
Grammar of Graphics: Implementation

- Wilkinson illustrates the GoG with a programming language (GPL: the Graphics Production Language)
- GPL statements
  - **DATA**: expressions that create variables to display from data sets
  - **TRANS**: variable transformations prior to plotting (e.g., ranking the data points)
  - **ELEMENT**: define graphical elements (e.g., points, lines, ...) and their aesthetic attributes (e.g., shape, color, ...) to use in the display
  - **SCALE**: apply scale transformations to the plot (e.g., square root or log)
  - **COORD**: select the coordinate system for use in the graphic (e.g., Cartesian, polar)
  - **GUIDE**: guides to aid interpretation (axes, legends)

GPL example: scatterplot

A simple scatterplot of the Iris data, points colored by species

```plaintext
DATA: x = "SepalLength"
DATA: y = "SepalWidth"
DATA: z = "Species"
TRANS: x = x
TRANS: y = y
ELEMENT: point(position(x*y), color(z))
COORD: rect(dim(1,2))
SCALE: linear(dim(1))
SCALE: linear(dim(2))
GUIDE: axis(dim(1), label("Sepal Length"))
GUIDE: axis(dim(2), label("Sepal Width"))
```

A smoothed contour plot of birth rate vs. death rate for selected countries

```plaintext
ELEMENT: point(position(birth*death), label(country))
ELEMENT: contour(position(smooth.kernel.density(birth*death)), color.hue())
GUIDE: form.line(position((0,0), (30,30)), label("Zero population growth"))
GUIDE: axis(dim(1), label("Birth rate"))
GUIDE: axis(dim(2), label("Death rate"))
```

GPL syntax

The essential features of a graph are described by **ELEMENT**
- Within this, the geometrical objects (point, line, interval, ...) are given
- Within this, their visual properties (position, color) and statistical summaries are given

Some typical graph types:

- **scatterplot**
  ```plaintext
  ELEMENT: point(position (x*y))
  ```
- **line chart**
  ```plaintext
  ELEMENT: line(position (x*y))
  ```
- **bar chart**
  ```plaintext
  ELEMENT: interval(position (x*y))
  ```
- **hor. bar chart**
  ```plaintext
  COORD: rotate (270)
  ELEMENT: point(position (x*y))
  ```
- **clustered bar chart**
  ```plaintext
  ELEMENT: interval.dodge(position (x*y), color (c))
  ```
- **stacked bar chart**
  ```plaintext
  ELEMENT: interval.stack(position (summary.proportion (y), color (c)))
  ```
- **histogram**
  ```plaintext
  ELEMENT: interval(position (summary.count (bin.rect (y))))
  ```

Facets & frames

Tables of graphs:
- Facets: → graphs of subset
- Frames: → separate graphs

Linked micromap:
- Population density of US, divided in octiles
- States in each octile shown separately

Colorless green graphs sleep furiously

- JSM 2017: Dinner with Lee Wilkinson, Howard Wainer, Paul Vellman, & others
- The great debate:
  - LW: The GoG is a complete theory, a formal mathematical model comprehending all graphs.
  - "Beauty is truth, truth beauty."—that is all Ye know on earth, and all ye need to know.
  - MF: There is more--
    - Implementation matters: translating a graphic idea into a finished graph should be facilitated by the language of graphic code.
    - A productive language for graphs should encompass the steps of data analysis
  - Pere Milán: A truly expressive graphic language should recommend the right graphic(s) to “get the message home”

Wickham: ggplot2

- ggplot2: Elegant graphics for data analysis
  - a computational language for thinking about & constructing graphs
  - sensible, aesthetically pleasing defaults
    - + themes: default, bw, journal, tufte, ...
  - infinitely extendable
    - ggplot extensions: http://www.ggplot2-exts.org/

- Implementation of GoG in R as layers of a graphic
  - Basic layers:
    - Data,
    - Aesthetics (data → plot mapping)
    - Geoms (points, lines, bars, ...),
  - Statistics: summaries & models
  - Coordinates: plotting space
  - Facets: partition into sub-plots
  - Themes: define the general features of all graphical elements
Every graph can be described as a combination of independent building blocks, connected by “+” (read: “and”)

- **data**: a data frame: quantitative, categorical; local or database query
- **aesthetic mapping of variables into visual properties**: size, color, x, y
- **geometric objects (“geom”)**: points, lines, areas, arrows, ...
- **coordinate system (“coord”)**: Cartesian, log, polar, map,

```
ggplot(FMA, aes(x=F, y=A, color=F, size=A) + geom_point())
```

In this call:
- **data=mtcars**: data frame
- **aes(x=, y=)**: plot X,Y variables
- **aes(color=, shape=)**: attributes
- **+ geom_point():** what to plot
- the coordinate system is taken to be the standard Cartesian (x,y)
- a corresponding legend is automatically generated

Wow! I can really see something there.

How can I enhance this visualization?

Easy: add a `geom_smooth()` to fit linear regressions for each level of cyl

```
ggplot(mtcars, aes(x=hp, y=mpg, color=cyl, shape=cyl)) + geom_point(size=3) + geom_smooth(method="lm", aes(fill=cyl))
```

The implementation of GoG ideas in ggplot2 for R created a more expressive language for data graphs

- **layers**: graph elements combined with “+” (read: “and”)
- **themes**: change graphic elements consistently
ggplot2: more geoms

ggplot2 facilitates graphical thinking by making a clear separation among:
- mapping data variables to plot features (aes());
- geometric objects (geom_());
- statistical summaries (stat_());

ggplot2: layers & aes()

Aesthetic attributes in the ggplot() call are passed to geom_() layers.
Other attributes can be passed as constants (size=3, color="black") or with aes(color=, ...) in different layers.
This plot adds an overall loess smooth to the previous plot.

ggplot2: themes

All the graphical attributes of ggplot2 are governed by themes – settings for all aspects of a plot.
A given plot can be rendered quite differently just by changing the theme.
If you haven’t saved the ggplot object, last_plot() gives you something to work with further.

ggplot2: facets

Facets divide a plot into separate subplots based on one or more discrete variables.
ggplot2: extensions

ggplot2 provides a prototype system for implementing new geoms, stats, themes, ...
Many of these are listed at http://www.ggplot2-exts.org/gallery/

A larger view: Data science

- Data science treats statistics & data visualization as parts of a larger process
  - Data import: text files, data bases, web scraping, ...
  - Data cleaning → “tidy data”
  - Model building & visualization
  - Reproducible report writing

The tidyverse of R packages

Import → Tidy → Transform → Visualise → Communicate

Import → Model

Visualise → ggplot2 → ggmap

Communicate
Summary

• Graphical developers in the Golden Age recognized the idea of “graphic language,” but could not define it.
• Bertin first formalized the relations between graphical features (“retinal variables”), data attributes (O, Q, ≠, ≡), and “reading levels”
• Wilkinson, in GoG, created a comprehensive syntax and algebra to define any graph
• Wickham, in ggplot2, created an expressive language to ease the translation of graphic ideas into plots.