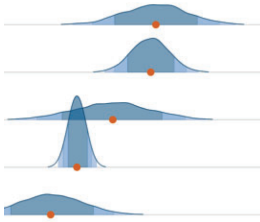
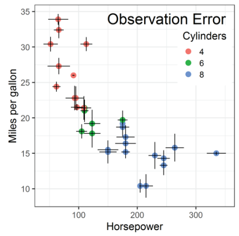




Visualizing Uncertainty

Michael Friendly

Psych 6135

http://euclid.psych.yorku.ca/www/psy6135/

Topics

- Problems with uncertainty in visualization
- Visualizing distributions
- “Error bars”
- Uncertainty in fitted curves
- Hypothetical outcome plots
- Cartographic uncertainty

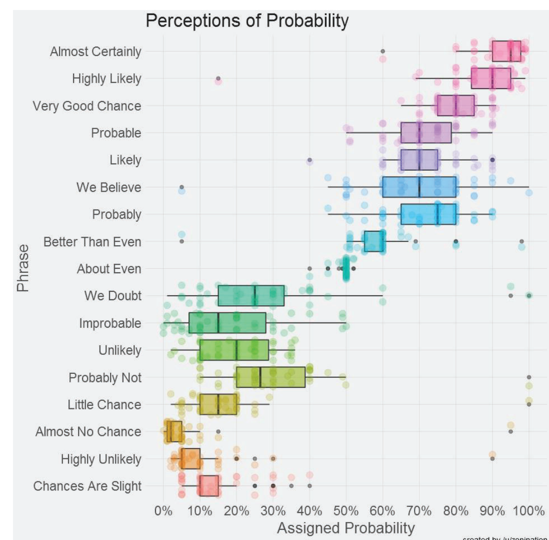
Problems

- Uncertainty is fundamental to data analysis & models
  - data: IQR, std dev., std error, … (variation)
  - models:
    - classical: confidence intervals, p-values;
    - Bayesian: credible intervals, posterior distributions
- In data graphics,
  - Easy to show “fit” – means, regression estimates, …
  - Harder to show the uncertainty in these numbers

How people view “probability”

What makes this graph successful?

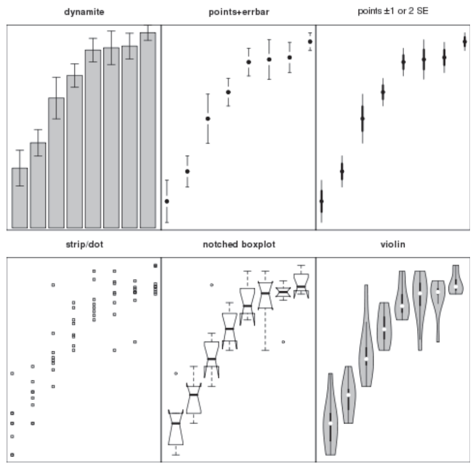
Note the wide range of variability (uncertainty) in the estimates:
“about even” vs. “we believe”



Comparing groups: Summary + Uncertainty

Six different graphs for comparing groups in a one-way design

- which group means differ?
- equal variability?
- distribution shape?
- what do error bars mean?
- unusual observations?



Never use dynamite plots

Always explain what error bars mean

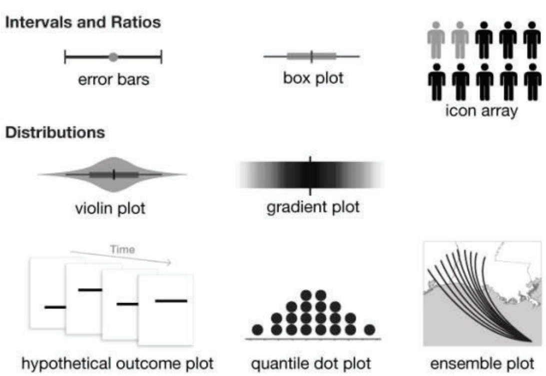
Consider tradeoff between summarization & exposure

Don't dynamite me!

rage

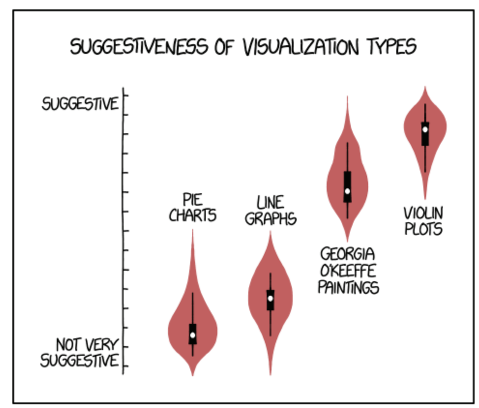
joy

Graphical annotations for uncertainty



From: Padilla, Kay & Hullman (2021), *Uncertainty Visualization*, DOI: 10.1002/9781118445112.stat08296

Visualizing distributions

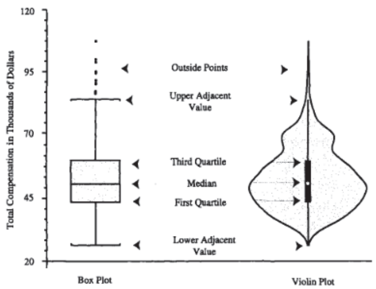


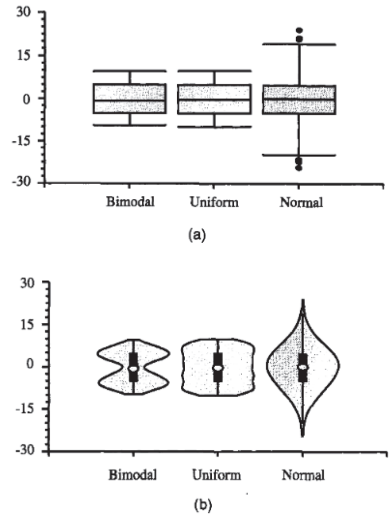
From: https://xkcd.com/1967/

Violin plots

Boxplots are great for ~ normal data
- Shows center, spread, outliers

Violin plots add a (reflected) density curve to show the shape of the distribution

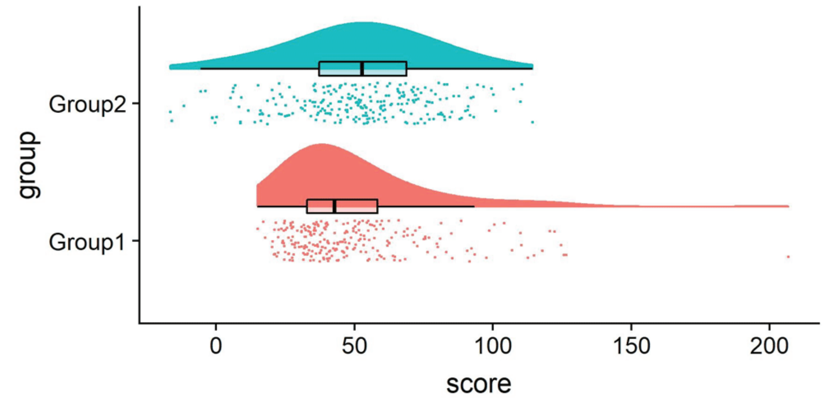




Hintze & Nelson (1998), *American Statistician*, 52:2, 181-184

Raincloud plots

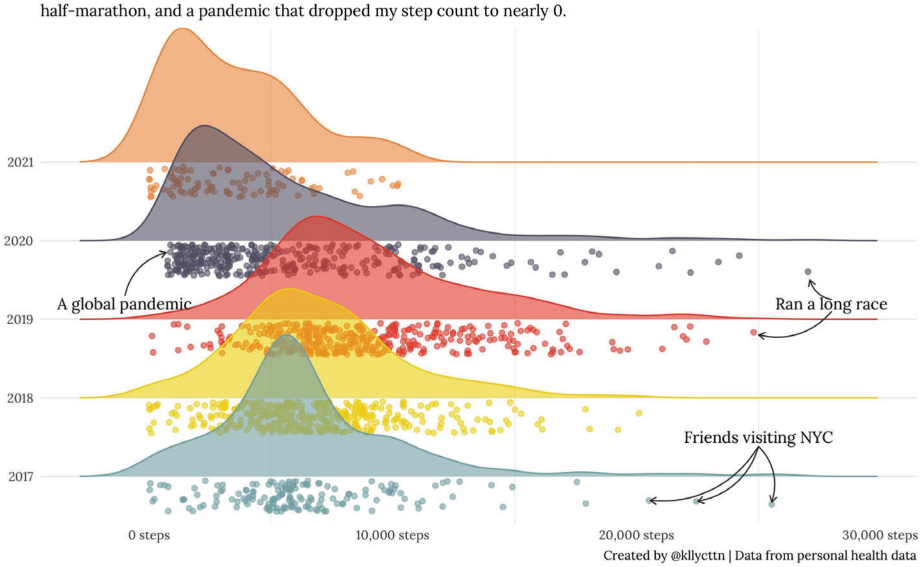
Raincloud plots are similar, but also show the observations as jittered points



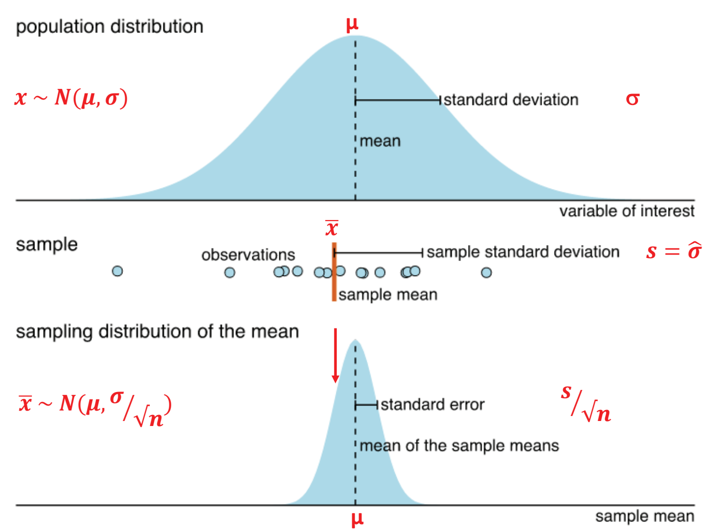
Allen M, Poggiali D, Whitaker K et al. Raincloud plots: a multi-platform tool for robust data visualization [version 2].
Wellcome Open Res 2021, 4:63 (doi: 10.12688/wellcomeopenres.15191.2)

How many steps have I taken since 2017?

Since July 2017, I have tracked the number of steps I've taken (almost) every day. In a little over 4 years, I have taken **9,232,798** steps. This includes days spent walking around New York with visiting friends, running a half-marathon, and a pandemic that dropped my step count to nearly 0.



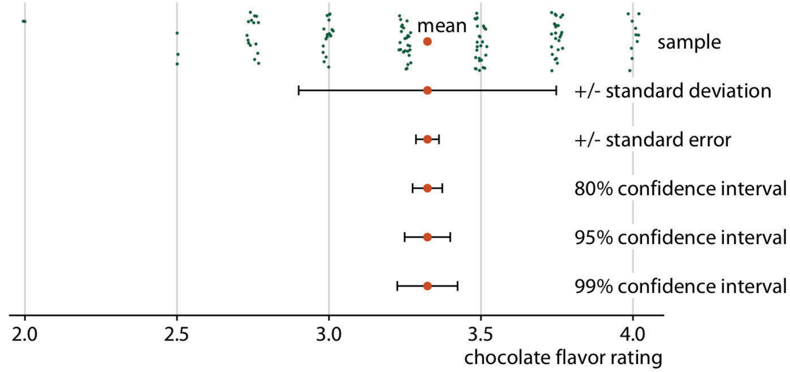
Key ideas of σtatistical σampling



From: Claus Wilke (2021), *Fundamentals of Data Visualization*, https://clauswilke.com/dataviz/ , Ch 16

Visualizing distributions: Error bars

There are many ways to show variability in a single sample

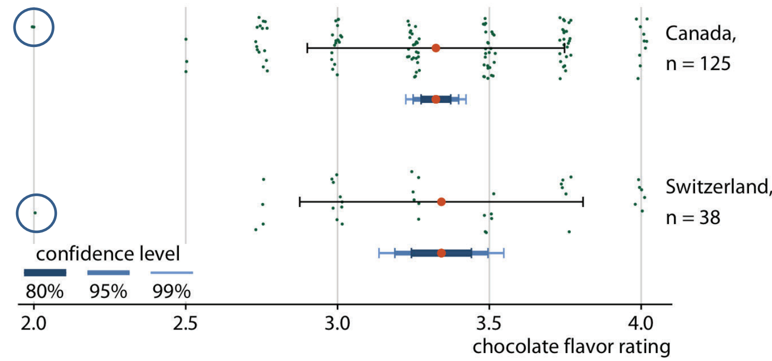


Expert ratings of 125 chocolate bars manufactured in Canada

From: Claus Wilke (2021), *Fundamentals of Data Visualization*, https://clauswilke.com/dataviz/ , Ch 16

Comparing distributions: Sample size

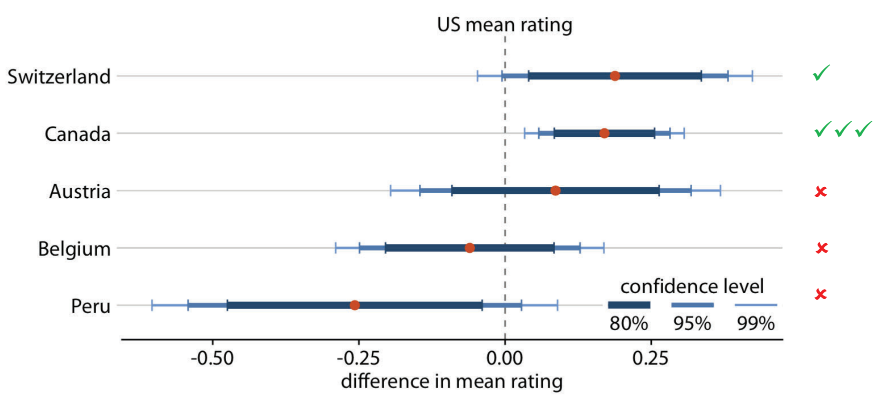
- means and standard deviations are similar for Canada & Switzerland
- confidence interval widths ~ $1/\sqrt{n}$
- can show different sized confidence bands together 
- dots show the data: are there any outliers?



From: Claus Wilke (2021), *Fundamentals of Data Visualization*, https://clauswilke.com/dataviz/ , Ch 16

Comparing distributions: Contrasts

- For comparison of one group to all others, plot the difference directly
- Easy to see which differences exclude 0, at what confidence level



From: Claus Wilke (2021), *Fundamentals of Data Visualization*, https://clauswilke.com/dataviz/ , Ch 16

What kind of intervals?

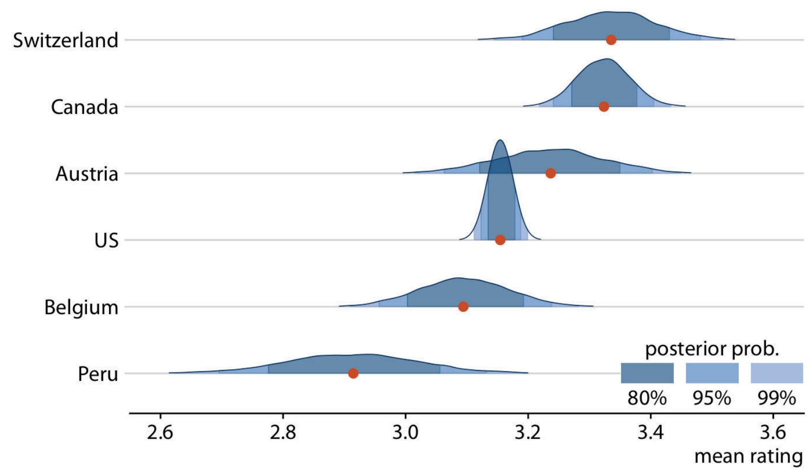
Frequentist

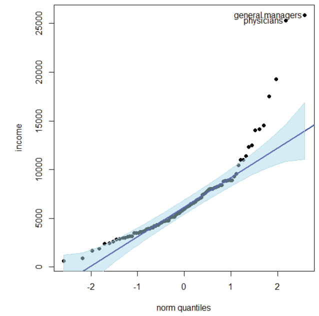
- Confidence interval
- Scope: repeated (hypothetical) samples
- Center: parameter estimate
  - $\mu \rightarrow \bar{x};\ \beta \rightarrow \hat{\beta}$
- Width: ~ std. error= $\hat{\sigma}/\sqrt{n}$
- Interpretation: true parameter w/in this interval 1-α %

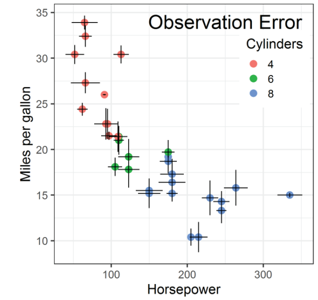
Bayesian

- Credibility interval
- Scope: repeated draws from the posterior distribution
- Center: median of posterior distribution
- Width: MAD sd of posterior
- Interpretation: Given prior, expect parameter w/in this interval 1-α %

Bayesian intervals







- QQplots
- Model fit plots

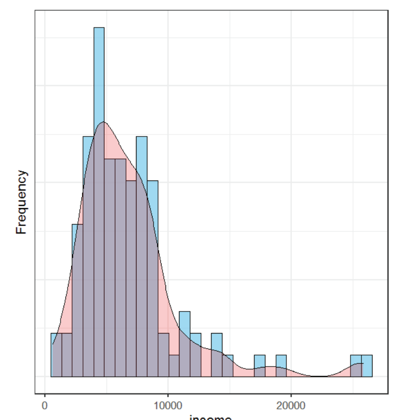
Uncertainty in fits & curves

QQ plots

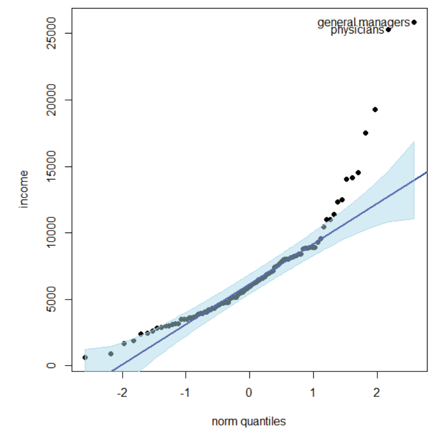
- How close is my data to a {Normal | exponential | χ2} distribution?
- There are lots of statistical tests, but these don't tell why or where a distribution is rejected.
- These tests are also overly sensitive to small departures
- Plot observed Quantiles vs. theoretical Quantiles
  - If observed ~ theoretical with slope = 1, OK
  - Confidence bands help to identify outliers
- Use cases:
  - Is a single variable reasonably normally distributed?
  - Are the residuals from my linear model Normal?
  - Outliers in multivariate data? $D^2 \sim \chi 2 \rightarrow$ chisq QQ plot

Prestige data: income

Income is clearly positively skewed.
(But normality is not required for predictors.)



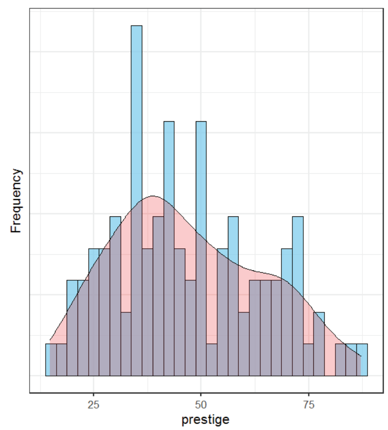
This shows up as a U-shaped pattern
The 95 % confidence band shows greatest departure in the upper tail

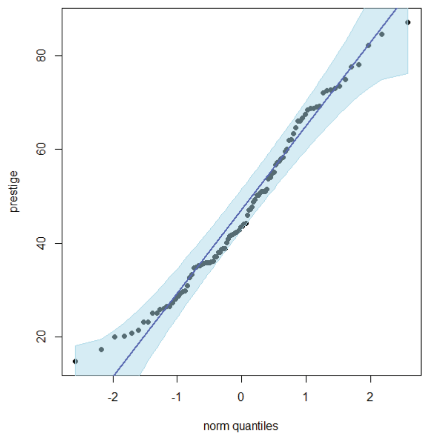


Prestige data: prestige

Occupational prestige doesn't look precisely normal, but not that bad.

The 95% confidence band includes all the observations





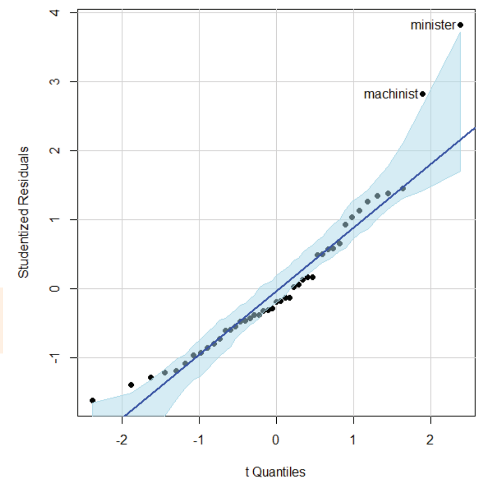
Prestige data: residuals

Normality of residuals is more important for linear models

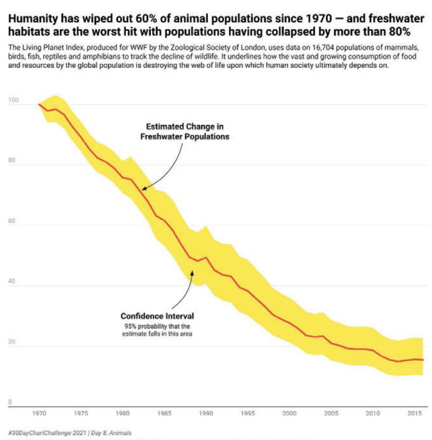
Some small evidence of + skew

Confidence bands help to identify potential outliers – badly fitted pts

```
qqPlot(lm(prestige ~ income + education + type, data=Duncan))
```



Curves + Uncertainty



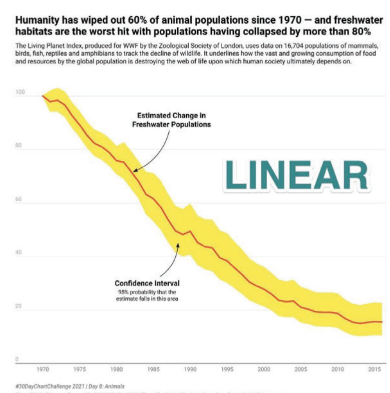
Cederic Scherer used this graphic to argue about the decline of animal & freshwater populations.

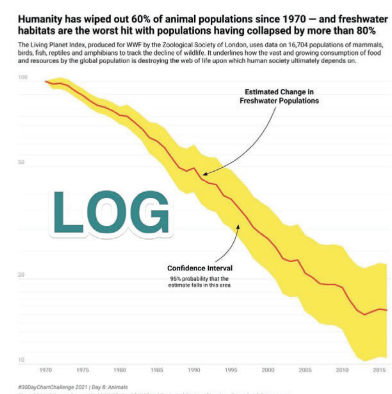
Details aside, the confidence band gives visual evidence that the decline is systematic.

From: https://twitter.com/CedScherer/status/1380211291466399744

Curves + Uncertainty: Scale

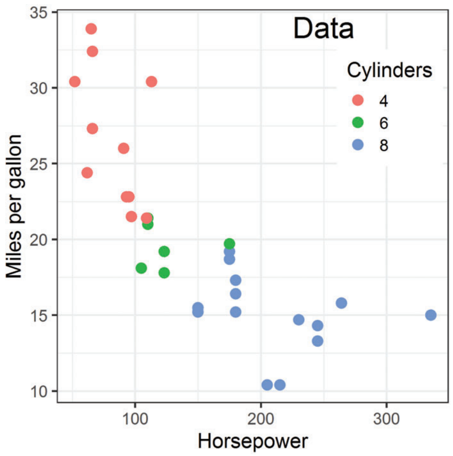
Arguably, percent reduction in animal population should be viewed on a log scale. Transformed uncertainty intervals are here the logs of the Upper/Lower levels





Fitted curves

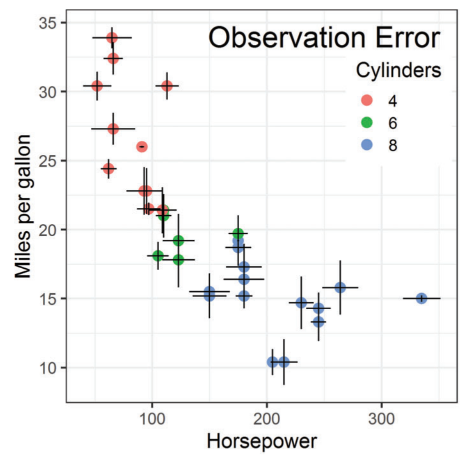
Data on gas mileage of *Motor Trend* 1974 cars



Sources of uncertainty:

- Observations: **measurement error** in MPG and/or HP?
- **Model form**: Linear? Quadratic? Interaction with cylinders
- **Model fit uncertainty**: normal theory CIs? Bootstrap? Bayesian?

Measurement uncertainty



Sometimes, we can quantify the uncertainty (“error”) in values of *x* and or *y*.
e.g., each point is the average of n>1 cars.

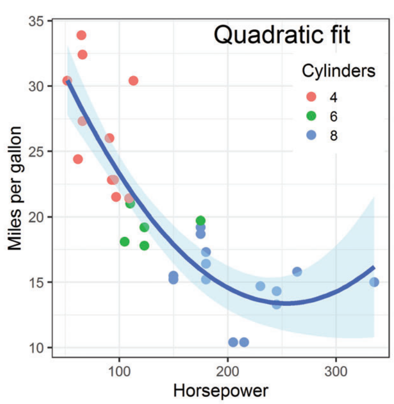
Fitted models allow for errors in *y*:
y = f(*x*) + error
and find estimates to minimize error

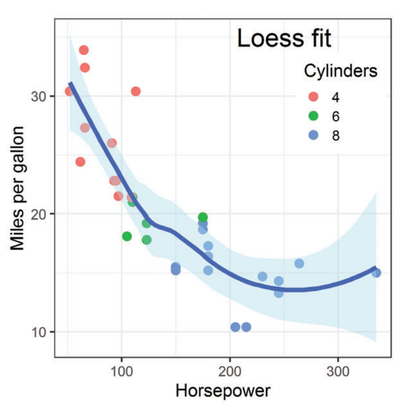
Most fitted models assume *x* is measured w/o error.

Big problem if error ~ f(x, other xs)

Model forms: nonlinear fits

When a relation is clearly non-linear, we can fit alternative models.
The CI bands tell us where the data is too thin to rely on the predicted value.

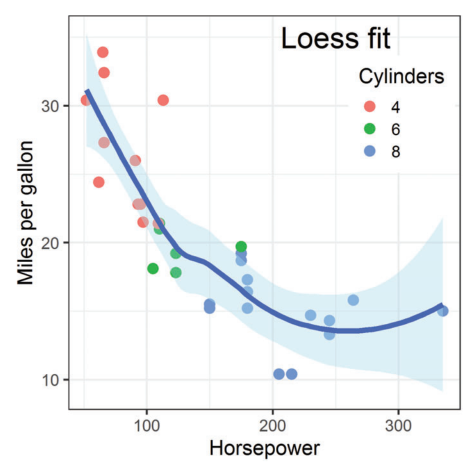





```
p1 + geom_smooth(method = lm, formula = y~poly(x,2), ...)
```

```
p1 + geom_smooth(method = loess, formula = y~x, ...)
```


Fitted curves: smoothers

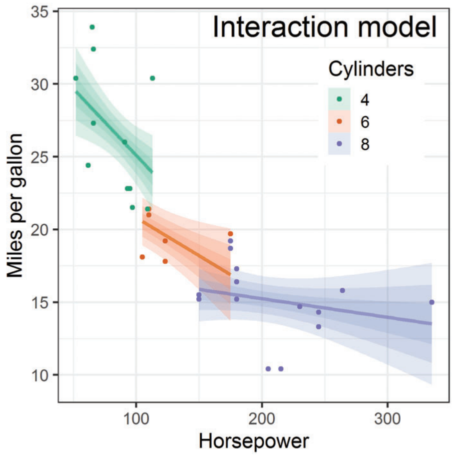


In each case, the confidence band gives visual evidence for uncertainty of the predicted values.

But, uncertainty may be expressed differently.

- a formula for std. error based on normal/large sample theory
- envelope of (normal) simulations
- Bayesian predictive distribution

Interaction models

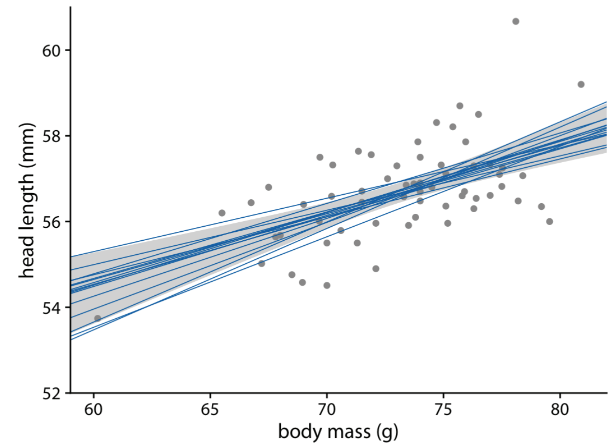


The non-linear relation between hp & mpg can (arguably) be better explained by a model that allows different slopes for 4, 6, 8 cylinders.

The graph shows normal theory CIs at 95%, 90%, and 80% for each cylinder level

Simulations to convey uncertainty

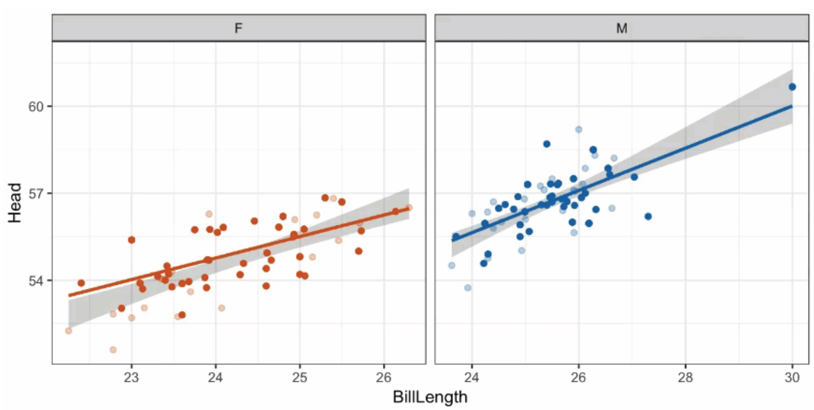
Simulating fits from the data (e.g., bootstrap, Bayesian estimation) shows the variability. Doesn't rely on classical, normal theory.



Animation to understand uncertainty

All assessments of uncertainty rely on a comparison: data vs. could have been
- Sampling distributions, simulations, Bayesian posterior distributions, …

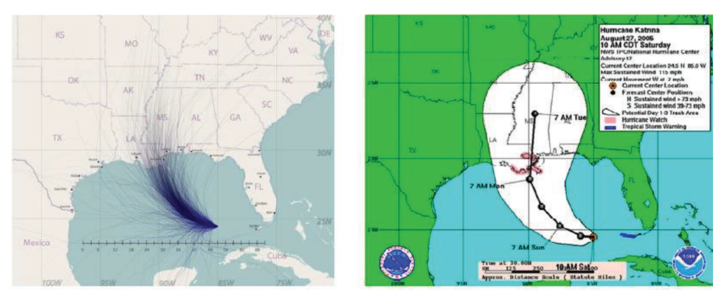
Sometimes useful to appreciate the variability with animated graphics



Geographic uncertainty

Predicting the path of hurricanes:

- Given what we can measure today (location, wind speed, direction, …) where is this hurricane likely to be in 1 day, 3 days, 5 days?
- Most forecasts are based on an ensemble of predictions, representing the uncertainty in initial conditions, model physics, …
- Often this is represented as a "cone of uncertainty"

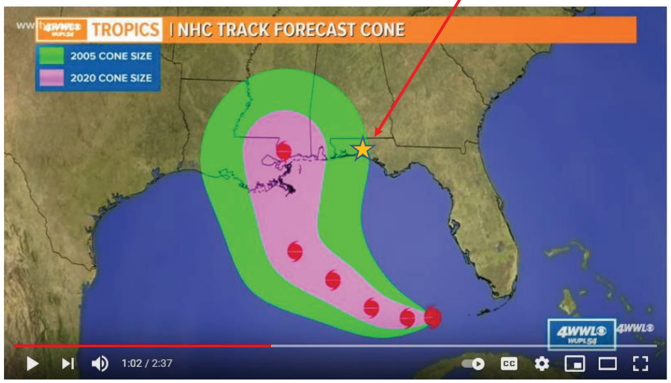


(a) Storm path ensemble

(b) Uncertainty cone.

What is the Cone of Uncertainty?

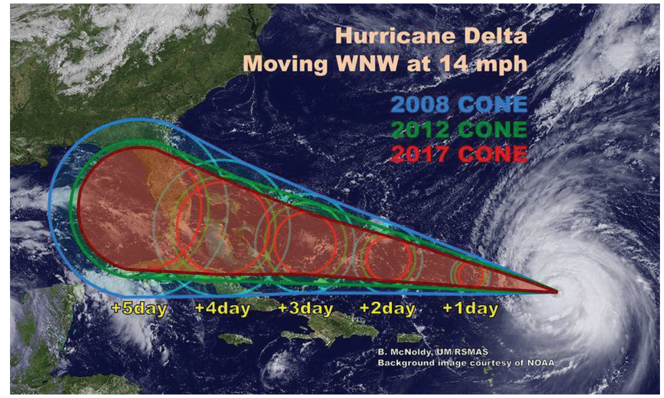
As seen on TV:
- The center is meant to track the average prediction, either over models or history
- The cone size generally represents some “2/3 confidence interval”
- Does this mean I am safe if I lived in Tallahassee FL ☆ in 2005? 2020?



From: https://www.youtube.com/watch?v=nE6XaHtpm04

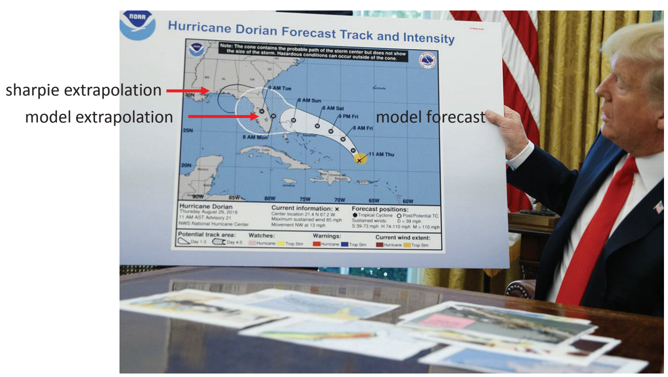
The Incredible Shrinking Cone

Changes in presumed accuracy are often shown as below
- The cone represents the probable track of the center of a tropical cyclone, formed by enclosing the area swept out by a set of circles along the forecast track (at 12, 24, 36 hours, etc).
- The size of each circle is set so that two-thirds of historical official forecast errors over a 5-year sample fall within the circle.



Sharpiegate

In Sept. 2019, Donald Trump went live with “extrapolated” predictions of the path of Hurricane Dorian.
- He had earlier predicted it would hit Alabama & Georgia.
- Let it be said, let it be written (with a sharpie)



From: https://www.theguardian.com/us-news/2019/sep/05/trump-hurricane-dorian-alabama-map-sharpiegate