Eigenvalues and eigenvectors with R

The goal of this exercise is to introduce a few ideas of eigenvalues and eigenvectors with R.

1. Start R Studio in the usual way, and load the `matlib` library

   ```
   library(matlib)
   ```

   Enter the following matrix, **A**, considered to be the variance-covariance matrix of a sample.

   $$A = \begin{pmatrix} 13 & -4 & 2 \\ -4 & 11 & -2 \\ 2 & -2 & 8 \end{pmatrix}$$

2. Use the `eigen()` function to find the eigenvalues and eigenvectors. Note that it returns a list, (values, vectors) and it is handy to assign these to separate names.

   ```
   ev <- eigen(A)
   # extract components
   values <- ev$values
   vectors <- ev$vectors
   ```

3. Verify that (a) the trace of **A** = sum of a_{ii} = sum of eigenvalues

   ```
   tr(A)
   sum(values)
   ```

   R hints for the following:

   - `sum(X)` sums the elements of a vector or matrix;
   - `tr(A)` and `sum(diag(A))` gives the sum of diagonal elements of a matrix;
   - `prod(X)` gives the product of elements of a vector;
   - `sum(X^2)` give the sum of squares of a vector or matrix.
   - `zapsmall(X)` makes very tiny numbers 0

4. Verify also that (b) the determinant of A = product of eigenvalues; (c) the sum of squares of the elements of **A** = sum of squares of eigenvalues.

5. Find the rank of the matrix **A**, using either the rank function, `R(A)`, or by inspection, from `echelon(A)`. How does this relate to `det(A)` ?

6. Find the eigenvalues and eigenvectors of the matrix $\mathbf{A}^{-1}$. How do they relate to the corresponding values for **A** itself? What about the eigenvalues and vectors of $\mathbf{A}^2 = \mathbf{A} * \mathbf{A}$? What is the general rule?

7. Show that the matrix V of eigenvectors is orthonormal, i.e., `t(V) * V` = I.

8. Find the product `t(V) * A * V`. What is the result?

9. Find each of the following products of a column of V with the corresponding element of L.

```
L = values
V = vectors
A1 = L[1] * V[,1] %*% t(V[,1])
A2 = L[2] * V[,2] %*% t(V[,2])
A3 = L[3] * V[,3] %*% t(V[,3])
```

Show that:

- `A = A1 + A2 + A3`
- `sum(A1^2) = L[1]^2`, and so on for the others
- each of A1, A2, A3 is of rank = 1