Eigenvalues and eigenvectors

The goal of this exercise is to introduce a few ideas of eigenvalues and eigenvectors in SAS/IML. It follows the example in http://www.psych.yorku.ca/friendly/lab/files/psy6140/examples/iml/imleig.sas

1. Start SAS/IML in the usual interactive way, and load the `matlib` library

   ```
   ods listing;
   proc iml;
     reset print log fuzz fw=5;
     %include iml(matlib);
   ```

   Enter the following matrix, **A**, considered to be the variance-covariance matrix of a sample.

   $$A = \begin{pmatrix} 13 & -4 & 2 \\ -4 & 11 & -2 \\ 2 & -2 & 8 \end{pmatrix}$$

2. Use the eigen function to find the eigenvalues and eigenvectors.

   ```
   call eigen(L, V, A);
   ```

   You can also use the functions `eigval(A)` and `eigvec(A)` to find eigenvalues and eigenvectors separately. Try them:

   ```
   L = eigval(A);
   V = eigvec(A);
   ```

3. Verify that (a) the trace of **A** = sum of a_{ii} = sum of eigenvalues; (b) the determinant of A = product of eigenvalues; (c) the sum of squares of the elements of **A** = sum of squares of eigenvalues.

   IML hints:
   - `sum(X)` and `X[+]` sum the elements of a vector or matrix;
   - `sum(vecdiag(A))` gives the sum of diagonal elements of a matrix;
   - `X[#]` gives the product of elements of a vector;
   - `ssq(X)` and `sum(X##2)` give the sum of squares of a vector or matrix.

4. Find the rank of the matrix **A**, using either the rank function, `r(A)`, or by inspection, from `echelon(A)`. How does this relate to det(A) ?

5. Find the eigenvalues and eigenvectors of the matrix $\mathbf{A}^{-1}$. How do they relate to the corresponding values for **A** itself? What about the eigenvalues and vectors of $\mathbf{A}^2 = \mathbf{A} * \mathbf{A}$? What is the general rule?

6. Show that the matrix V of eigenvectors is orthonormal, i.e., `t(V) * V` = I.

7. Find the product `t(V) * A * V`. What is the result?

8. Find each of the following products of a column of V with the corresponding element of `L`.

```
A1 = L[1] # V[,1] * t(V[,1]);
A2 = L[2] # V[,2] * t(V[,2]);
A3 = L[3] # V[,3] * t(V[,3]);
```

Show that:

- A = A1 + A2 + A3
- `ssq(A1) = L[1]##2`, and so on for the others
- each of A1, A2, A3 is of rank = 1